Parameterized Complexity for Graph Linear Arrangement Problems

Gregory Gutin
Royal Holloway, University of London and University of Haifa
(joint work with A. Rafiey, S. Szeider and A. Yeo)

A linear arrangement (LA) of a graph $G = (V, E)$ of order $n$ is a bijection $\alpha : V \rightarrow \{1, \ldots, n\}$. The length (net length) of an edge $uv \in E$ relative to $\alpha$ is defined as $\lambda_{\alpha}(uv) = |\alpha(u) - \alpha(v)|$ ($\lambda_{\alpha}(uv) = |\alpha(u) - \alpha(v)| - 1$). The cost (net cost) of an LA $\alpha$ is is the sum of the costs (net costs) relative to $\alpha$ of edges of $G$. For an LA $\alpha$ of $G = (V, E)$, its profile is $\text{prf}(G) = \sum_{v \in V} \max\{\alpha(v) - \alpha(u) : u \in N[v], \alpha(u) \leq \alpha(v)\}$.

In his recent Habilitation thesis and at his talk at Dagstuhl in July 2005, H. Fernau considers the following problem: given a graph $G = (V, E)$ and a parameter $k$, check whether there is an LA of net cost at most $k$. Fernau asks whether the problem is fixed-parameter tractable (FPT), i.e., whether it can be solved by an algorithm of time complexity $O(f(k)(|V| + |E|)^t)$, where $t$ is a constant independent of $k$ and $f$ is a computable function. We prove that the problem is FPT by deriving an algorithm of complexity $O(|V| + |E| + 5.88^k)$.

M. Serna and D.M. Thilikos (2005) ask whether the following problems are FPT: (i) given a graph $G = (V, E)$ and a parameter $k$, check whether there is an LA of cost at most $k|V|$; (ii) given a graph $G = (V, E)$ and a parameter $k$, check whether there is an LA of cost at most $k|E|$; (iii) given a graph $G = (V, E)$ and a parameter $k$, check whether there is an LA of profile at most $k|V|$. We prove that for any fixed $k \geq 2$ the following problems are NP-complete: check whether $G = (V, E)$ has an LA of cost at most $k|V|$ (cost at most $k|E|$, profile at most $k|V|$). Thus, the Serna-Thilikos problems are not FPT.