Abstract

A. Gyárfás: **A Nordhaus-Gaddum type inequality for the First-fit chromatic number**

A well known inequality (due to Nordhaus and Gaddum) relating the chromatic number of an n-vertex graph and its complement is $\chi(G)+\chi(G^c) \leq n+1$. In fact, the stronger inequality $col(G)+col(G^c) \leq n+1$ also holds - and I think it is easier to prove - where $col(G) = 1 + \max\{\delta(H) : H \subseteq G\}$ is the coloring- alias Wilf-Szekeres- number. There are results about the extension of this inequality to many-part decompositions and to other parameters (like Hadwiger number, list-chromatic number).

Zaker suggested to look at the analogous inequality for χ_{FF}, the First Fit chromatic number: the maximum number of classes in a partition of the vertex set of G into independent sets $A_1, \dots A_t$ so that for each $1 \leq i < j \leq t$, and for each $x \in A_j$ there exists $y \in A_i$ such that x, y are adjacent in G. Thus $\chi_{FF}(G)$ is the measure of the worst case behavior of the First-fit coloring on G. Note that $\chi_{FF}(G)$ and $col(G)$ are both between $\chi(G)$ and $\Delta(G)+1$, but they do not relate to each other.

In fact, Zaker conjectured that the Nordhaus-Gaddum inequality hardly changes, namely that for every n-vertex graph G, $\chi_{FF}(G) + \chi_{FF}(G^c) \leq n+2$. We show that the conjecture is true for bipartite graphs but in general

$$\left\lfloor \frac{5n}{4} \right\rfloor \leq \max\{\chi_{FF}(G) + \chi_{FF}(G^c) : |V(G)| = n\} \leq \left\lfloor \frac{5n+3}{4} \right\rfloor .$$

We extend the problem for multicolorings as well but our estimates do not give asymptotic for $k \geq 3$ colors.

The results are from Z. Füredi, A. Gyárfás, G. N. Sárközy, S. Selkow : Nordhaus-Gaddum type and other inequalities for the First-fit chromatic number (in preparation).