Branchwidth of graphic matroids.

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A branch-decomposition of a graph $G = (V, E)$ is a ternary tree $T$ and a bijection from the set of leaves of $T$ into the set of edges of $G$. Every edge $e$ of $T$ partitions $T \setminus e$ into two subtrees, and thus correspond to a bipartition $(E_1, E_2)$ of $E$, called $e$-separation. The width of $(E_1, E_2)$ is the number of vertices of $G$ incident to an edge of $E_1$ and an edge of $E_2$. The width of $T$ is the maximum width of an $e$-separation. Finally, the branchwidth of $G$ is the minimum width of a branch-decomposition of $G$.

The notion of branchwidth extends naturally to matroids, branch-decompositions being ternary trees which set of leaves is the ground set of the matroid. Here the width of a separation $(E_1, E_2)$ is $rk(E_1) + rk(E_2) - rk(E) + 1$, where $rk$ is the rank function of the matroid.

Answering a question of Geelen, Gerards, Robertson and Whittle, we prove that the branchwidth of a bridgeless graph is equal to the branchwidth of its cycle matroid.

Our result directly implies that the branchwidth of a planar bridgeless graph is equal to the branchwidth of its dual. This property was first proved by Seymour and Thomas.