L(p,q)-labelling of graphs

F. Havet*

Joint work with

J. van den Heuvel, C. McDiarmid, B. Reed and J.S. Sereni

An $L(p,q)$*-labellings of* G is an integer assignment f to the vertex set $V(G)$ such that : $|f(u)-f(v)| \geq p$, if $dist(u,v)=1$, and $|f(u)-f(v)| \geq q$, if $dist(u,v)=2$. The *span* of f is the difference between the largest and the smallest labels of f plus one. The $\lambda_{p,q}$*-number* of G, denoted by $\lambda_{p,q}(G)$, is the minimum span over all $L(p,q)$-labellings of G. Note that $L(1,0)$-labellings of G correspond to ordinary vertex colourings of G and $L(1,1)$-labelling of G to the vertex colourings of the square G^2 of G.

In 1992, Griggs and Yeh conjectured that $\lambda_{2,1}(G) \leq \Delta^2+1$. Diameter two cages such as the 5-cycle, the Petersen graph and the Hoffman-Singleton graph show that there exist graphs that in fact require Δ^2+1 colours, for $\Delta = 2,3,7$ and possibly one for $\Delta = 57$. The best upper so far was due to Gonçalves which shows $\lambda_{2,1}(G) \leq \Delta^2+\Delta-1$. With B. Reed and J.-S. Sereni we settle Griggs and Yeh conjecture for sufficiently large Δ.

Regarding planar graphs, far less colours suffice. In 1977, Wegner conjectured that $\lambda_{1,1}(G) = \chi(G^2) \leq \left\lfloor \frac{3}{2}\Delta \right\rfloor + 1$ if $\Delta \geq 8$ and gave examples showing that this bound would be tight. The asymptotically best known upper bound so far has been obtained by Molloy and Salavatipour. They show that for a planar graph G, $\lambda_{1,1}(G) \leq \left\lceil \frac{5}{3}\Delta \right\rceil + 78$. With J. van den Heuvel, C. McDiarmid and B. Reed, we show that $\lambda_{1,1}(G) \leq (1+o(1))\frac{3}{2}\Delta$.

These two results generalise to $L(p,q)$-labelling and list-colouring.

*Projet Mascotte, I3s (CNRS/UNSA) and INRIA, INRIA Sophia-Antipolis, 2004 route des Lucioles BP 93, 06902 Sophia-Antipolis Cedex, France