

Edge-critical subgraphs of Schrijver graphs

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The Kneser graph $KG(n, k)$ is the graph whose vertices are the k -element subsets of $\{1, \dots, n\}$, two k -subsets being adjacent if and only if they are disjoint. In 1978 Lovász proved that $KG(n, k)$ cannot be $(n - 2k + 1)$ -coloured, thus settling a longstanding conjecture of Kneser.

The Schrijver graph $SG(n, k)$ is the subgraph of $KG(n, k)$ induced by the k -subsets which contain no two consecutive elements in the cyclic order $(1, \dots, n)$. Schrijver sharpened Lovász's result by showing that $SG(n, k)$ cannot be $(n - 2k + 1)$ -coloured. He also showed that every proper induced subgraph of $SG(n, k)$ is $(n - 2k + 1)$ -colourable; in other words, Schrijver graphs are vertex-critical.

We will sharpen the result further by describing proper subgraphs of Schrijver graphs which cannot be $(n - 2k + 1)$ -coloured. Furthermore, we will show that they are edge-critical.

Joint work with Fabrício Benevides, Victor Campos and Tomáš Kaiser.