Total weight choosability of trees

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Abstract

A total-weighting of a graph $G = (V, E)$ is a mapping $f$ which assigns to each element $y \in V \cup E$ a real number $f(y)$ as the weight of $y$. A total-weighting $f$ of $G$ is proper if the colouring $\phi_f$ of the vertices of $G$ defined as $\phi_f(v) = f(v) + \sum_{e \ni v} f(e)$ is a proper colouring of $G$, i.e., $\phi_f(v) \neq \phi_f(u)$ for any edge $uv$. For positive integers $k$ and $k'$, a graph $G$ is called $(k, k')$-total-weight-choosable if whenever each vertex $v$ is given $k$ permissible weights and each edge $e$ is given $k'$ permissible weights, there is a proper total-weighting $f$ of $G$ which uses only permissible weights on each element $y \in V \cup E$. It is known that every tree is $(2, 2)$-total-weight-choosable and every tree other than $K_2$ is $(1, 3)$-total-weight-choosable. However, the problem of determining which trees are $(1, 2)$-total-weight-choosable remained open. In this talk, I present the result in a joint paper with Gerard Jennhwa Chang, Guan-Huei Duh and Tsai-Lien Wong, in which we solve this problem and characterizes all $(1, 2)$-total-weight-choosable trees. Based on this characterization, we give an algorithm that determines in linear time whether a given tree is $(1, 2)$-total-weight-choosable.