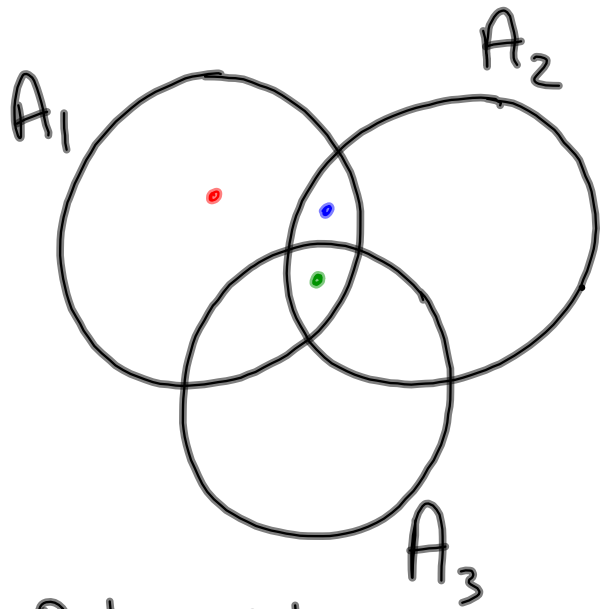


$$|A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3| - |A_1 \cap A_2| - |A_1 \cap A_3| - |A_2 \cap A_3| + |A_1 \cap A_2 \cap A_3|$$

okt 2-11:54

Thm 1 (Inclusion-Exclusion)

$$|A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n| =$$

$$\sum_{i=1}^{n} |A_i| - \sum_{1 \leq i < j \leq n} |A_i \cap A_j| + \sum_{1 \leq i < j < k \leq n} |A_i \cap A_j \cap A_k|$$

$$- \ldots + (-1)^{n+1} |A_1 \cap A_2 \cap \ldots \cap A_n|$$

P: Show that each element is counted precisely once right and left fix $1 \le r \le n$ and a in precisely r sets $A_{j_1}, \dots, A_{j_r}$

a is counted

$+$ $r = \binom{r}{1}$ times in $\sum |A_i|$

$-$ $\binom{r}{2}$ — " — $\sum |A_i \cap A_j|$

$+$ $\vdots$

$\binom{r}{m}$ — " — $\sum |A_{i_1} \cap \dots \cap A_{i_m}|$

$\vdots$

$1 = \binom{r}{r}$ — " — $|A_{j_1} \cap \dots \cap A_{j_r}|$

in total a is counted

$$\binom{r}{1} - \binom{r}{2} + \binom{r}{3} - \dots + (-1)^{r+1} \binom{r}{r}$$

$= 1$!! next page

$$0 = (1+(-1))^r = \sum_{k=0}^{r} \binom{r}{k} (-1)^k 1^{r-k}$$

$$0 = \binom{r}{0} - \binom{r}{1} + \binom{r}{2} \cdots$$

$0 = 1 -$ # counts for a in previous formula applications of I-E principle

- Count # elements out of N without given properties $P_1, P_2, \ldots, P_n$

denoted $N(P_1'P_2'\ldots P_n')$

from incl-excl:

$N(P_1'\ldots P_n') = N - |A_1 \cup A_2 \cup \ldots \cup A_n|$

where A_i = elements with P_i

$$N(P_1'\ldots P_n') = N - \sum N(P_i) + \sum N(P_iP_j) - \sum N(P_iP_jP_k) \ldots + (-1)^n N(P_1\ldots P_n)$$

- (

Ex1 p542 # integer (non-neg) s/s

1) $X_1+X_2+X_3=11$

2) $X_1\le 3, X_2\le 4, X_3\le 6$

Comment: without 2):

$****|**|*****$

$X_1=4$ $X_2=2$ $X_3=5$

$$\binom{11+3-1}{11}$$

including 2):

$P_1: X_1\ge 4$

$P_2: X_2\ge 5$

$P_3: X_3\ge 7$

Find $N(P_1'P_2'P_3')$

okt 2-12:44

Ex primes ≤ 100

$n = p \cdot q \Rightarrow \min\{p,q\} \leq \sqrt{n}$

only need to check for prime divisors ≤ 10

$2, 3, 5, 7$

P_1 $2|n$ P_2 $3|n$ P_3 $5|n$ P_4 $7|n$

Find $N(P_1' P_2' P_3' P_4')$

Erastotenes sieve

Finds primes $\leq B$ ← input

$L := \{2, 3, 4, \ldots, B\}$; $P := \emptyset$

while $L \neq \emptyset$ do

remove smallest element p from L

$P := P + p$

remove all multipla of p from L

$(p, 2p, 3p, \ldots)$

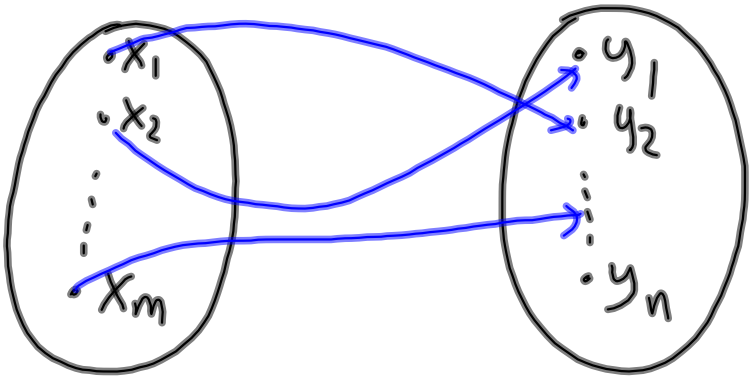
end

return P

$O(\sqrt{B})$ iterations

exponential in $\log_2 B$ ← input size

of onto fcts



n^m mappings in total
(not ness. onto)

P_i : y_i not hit

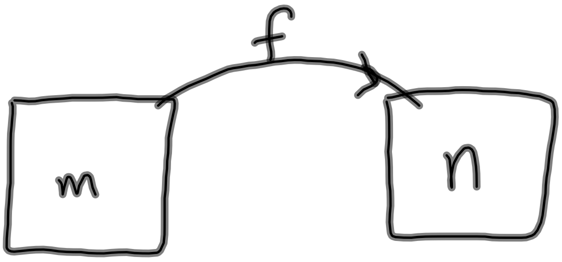
Find $N(P_1' P_2' \cdots P_n')$

Thm 1 # onto m-set → n-set
is

$$n^m - \binom{n}{1}(n-1)^m + \binom{n}{2}(n-2)^m$$
$$\cdots + (-1)^{n-1}\binom{n}{n-1} 1^m$$

okt 2-13:14

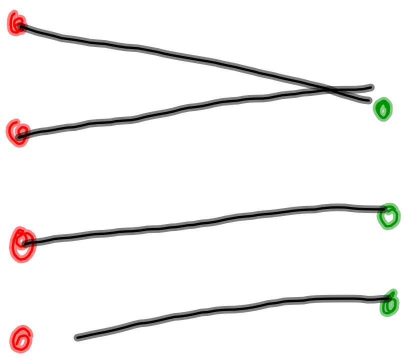
Onto functions $\Leftrightarrow$ Stirling nbs



Build f in two steps:

1. distribute m elemens in n boxes so that no box $\emptyset$

2. permute boxes



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