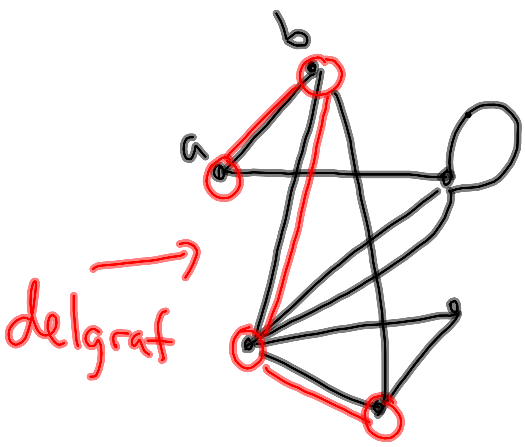
En graf $G=(V,E)$

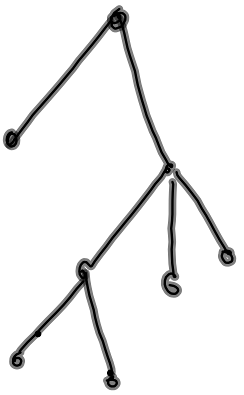
<u>Toft</u>

V punkterne i G — $P(G)=V$

(knuderne)

E kanterne i G — $K(G)=E$

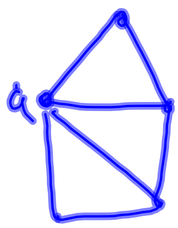




$E \subseteq V \times V$ — træ

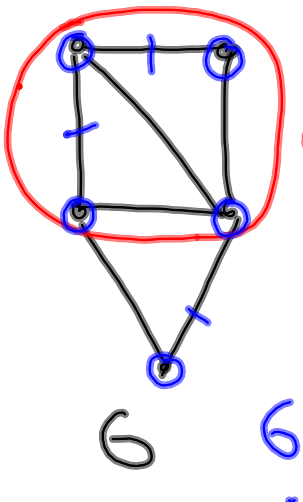
valensen (graden) af et punkt $x \in V$

$= \#$ Kanter med endepunkt i x



$v(a, G) = 4$

$d(a) = 4$

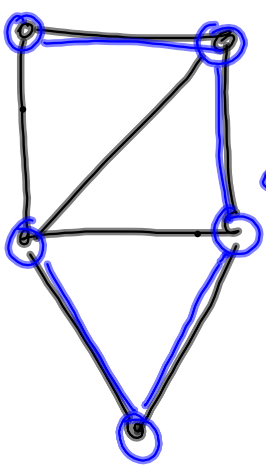


G' = alt inde i cirkel

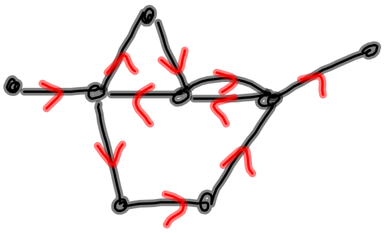
G' er induceret

G'' udspændende i G

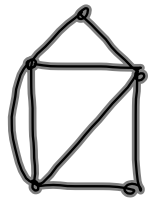
$P(G'') = P(G)$



udspændende træ (kredsløs)

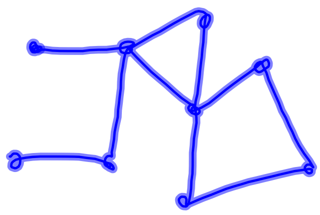


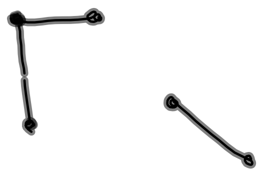
rute



lukket rute

Tur = rute uden kant gentagelse

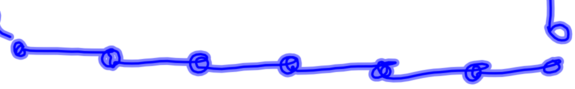




En vej (Path) =

rute uden pkt gentagelse

(a,b)-vej →



$a \neq b$

Kreds = 'lukket' vej $(a=b)$

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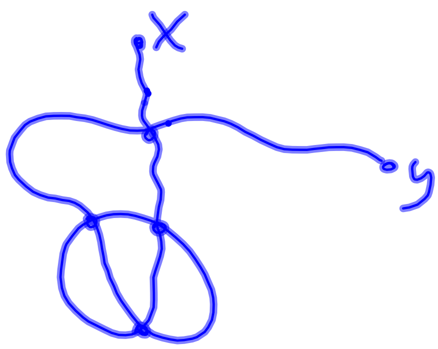
$G=(V,E)$ er sammenhængende

$\Updownarrow$ def

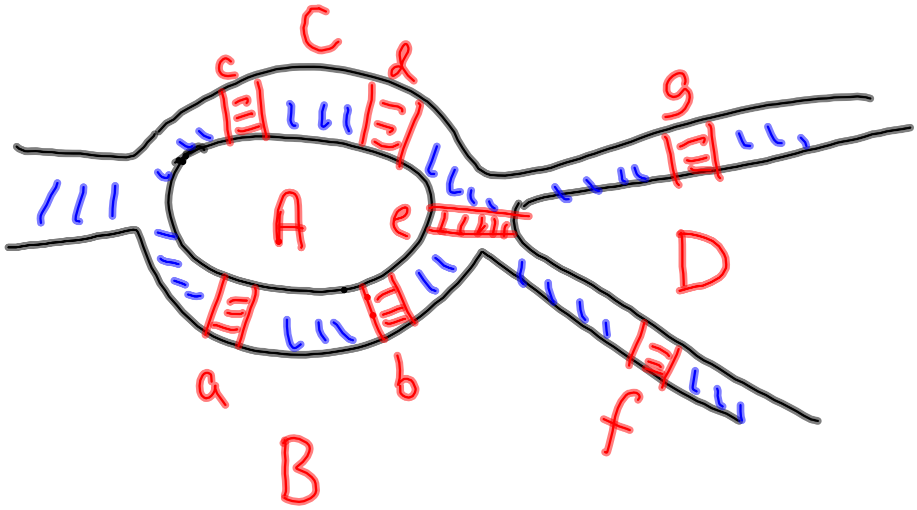
$\forall x,y \in P(G)\ \exists\ (x,y)$-vej

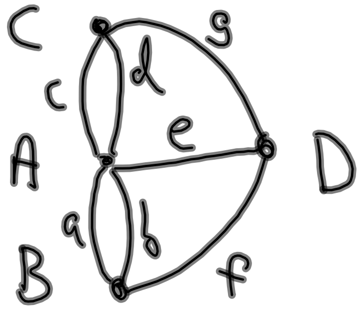
$\Updownarrow$

$\forall x,y \in P(G)\ \exists\ (x,y)$-rute



Korteste (x,y)-rute er en (x,y)-vej



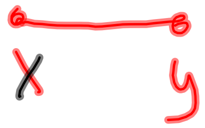


Euler-tur = tur som indeholder alle kanter i G

Sætn 1.1 $\forall G = (V,E)$ gælder

$$\sum_{x \in V} v(x,G) = 2|K(G)| = 2|E|$$

B:



Sætn. 1.2

Enhver graf har et lige antal pkt med ulige valens.

B: $$2|E| = \sum v(x,G)$$

$$= \sum_{x \text{ ulige val.}} v(x,G) + \sum_{x \text{ lige valens}} v(x,G)$$

lige

lige

lige

Sætn 1.3 Euler (1736)

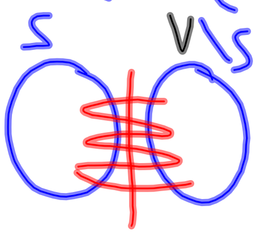
smh G har en lukket Eulertur

$\Updownarrow$

$v(x,G) \equiv 0 \mod 2 \;\; \forall v \in P(G)$

B: $\Downarrow$ Lad T være en lukket Eulertur

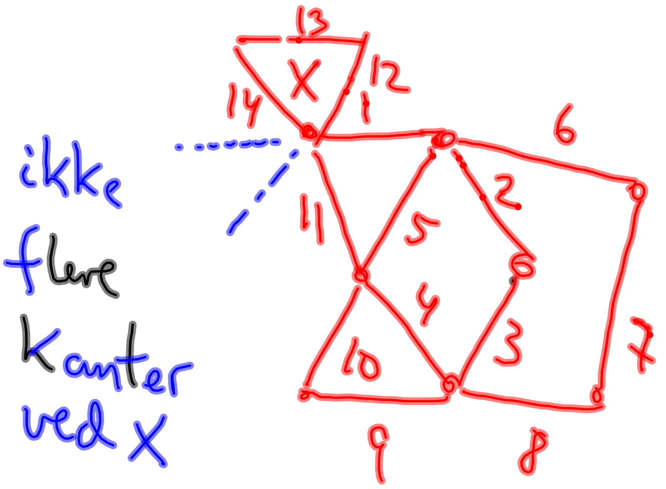
stærkere (faktisk ækvivalent!!)



T har lige antal kanter mellem S og $V \setminus S$

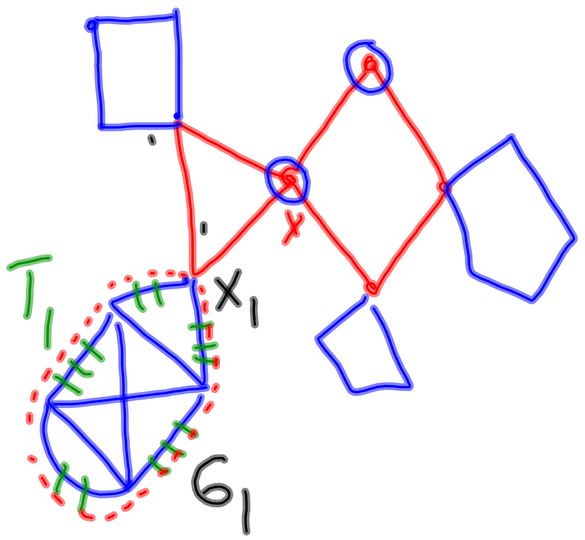
$\Uparrow$ Giver et konstruktivt bevis (bygger en Eulertur)

- vælg vilk. $x \in P(G)$
- Lav en maksimal tur T fra x:



- Hvis $T \cong K(G)$ ✓
- Ellers se på G'

$G' = G$ minus kanterne i T



G' har sm h komponenter $G_1, G_2, \dots, G_k$ $k>1$ hvor $G_k = \{x\}$

alle valenser i G_i er lige $(\forall i)$

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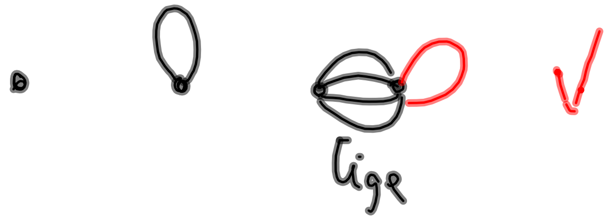
alternativt bevis: induktion over $n = |P(G)|$

<u>basis</u> $n \leq 2$



<u>ind. antagelse</u> sætningen gælder $\forall$ smh G' med $< n$ punkter.

Se forrige side!!

feb 9-13:37

bevis vha maksimalitet (modstrid)

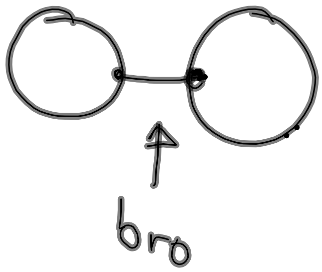
antag T er længst mulig (flest mulige kanter)

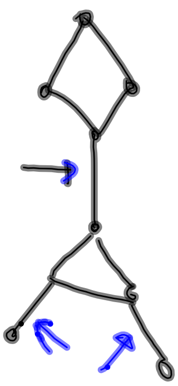
Vis at T er Eulertur

Se tidligere figur!

algoritme:

def

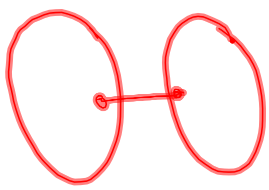




observation

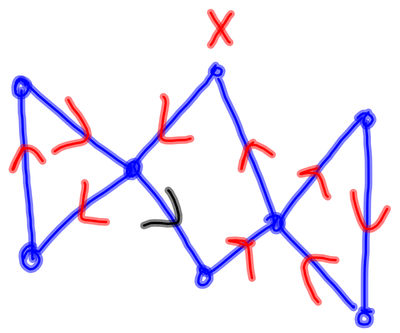
Hvis $v(x,G)$ lige $\forall x \in V(G)$

så har G ingen broer



<u>algoritmen</u> vælger nye Kant fra aktuelle pkt x_{i+1} ($x_1 - x_2 - x_3 - \ldots - x_i$) således, at (x_i, x_{i+1}) ikke er en bro i $G - \{(x_1,x_2), \ldots, (x_{i-1}, x_i)\}$

Hvis det er muligt



antag at T ikke er Eulertur $G' = G -$ kanter i T

$A = \{x \mid v(x, G') = 0\}$

$B = \{y \mid v(y, G') > 0\}$