Alternating Hamiltonian Paths in Edge-coloured complete graphs

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Abstract

We consider edge-coloured complete graphs K_n^c (where c denotes the number of colours). A trail (path,cycle) T is called alternating (properly coloured, PC), if any two adjacent vertices differ in colour. One of the typical problems in this field of graph theory is to get necessary and sufficient conditions for K_n^c to contain a Hamiltonian path or cycle. Seemingly the first appearance of alternating trails in edge-coloured graphs dates back to Petersen's famous paper (1891). Today, many results exist for 2-coloured graphs, but much less is known of the general case (for details, see the survey paper [2]). Bollobás and Erdös ([1]) posed the following (sharp) conjecture: Every K_n^c with $\Delta(K_n^c) < [n/2]$ has a PC Hamiltonian cycle (where $\Delta(K_n^c)$ denotes the maximal monochromatic degree). Häggkvist (1996) announced that every K_n^c satisfying $\Delta(K_n^c) < [n/2]$ contains a PC 2-factor. Bang-Jensen, Gutin and Yeo ([3]) proved that every K_n^c containing a PC 2-factor contains a PC Hamiltonian path. Together, these results give prove to a "weak" version of the conjecture of Bollobás and Erdös: Every K_n^c with $\Delta(K_n^c) < [n/2]$ contains a PC Hamiltonian path.

In this context Bang-Jensen, Gutin and Yeo ([3]) posed the following conjecture: K_n^c has a PC Hamiltonian path if and only if K_n^c contains a collection F consisting of a PC path P and a number of cycles $C_1, \ldots, C_t$ $(t \geq 0)$, each PC, such that the members of F are pairwise vertex disjoint and $V(P \bigcup C_1 \bigcup \ldots, \bigcup C_t) = V(K_n^c)$. They verified this conjecture for the case of two colours. In our paper we confirmed the conjecture for the general case (not depending on the number of colours).

Literatur

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[2] J. Bang-Jensen and G. Gutin, Alternating cycles and paths in edge-coloured multigraphs, *Discrete Math.* **165–166** (1997), 39–60.

[3] J. Bang-Jensen, G. Gutin and A. Yeo, Properly coloured Hamiltonian paths in edge-coloured complete graphs, *Discrete Appl. Math.* **82** (1998), 247–250.

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