Finding small k-arc-strong Spanning Subgraphs in k-arc-strong Tournaments

Jørgen Bang-Jensen * Jing Huang † **Anders Yeo** ‡

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Abstract

Given a k-arc-strong tournament T, we estimate the minimum number of arcs possible in a k-arc-strong spanning subgraph of T. We give a construction which shows that for each $k \geq 2$ there are tournaments T on n vertices such that every k-arc-strong spanning subgraph of T contains at least $nk + \frac{1}{8}(k+1)^2$ arcs. As our main result we prove that every k-arc-strong tournament contains a spanning k-arc-strong subdigraph with no more than $nk + 280k^2$ arcs. Our proof implies the existence of a polynomial algorithm which finds such a spanning k-strong subdigraph with at most $nk + 280k^2$arcs. We also discuss the implications of our results on related problems and conjectures.

*Department of Mathematics and Computer Science, University of Southern Denmark, Odense DK-5230, Denmark (email jbj@imada.sdu.dk).

†Department of Mathematics and Statistics, University of Victoria, Victoria, B.C., V8W 3P4, Canada (email: jing@math.uvic.ca). The second author wishes to acknowledge financial support from NSERC (under grant 203191-98).

‡Brics, Department of Computer Science, University of Aarhus, Ny Munkegade, building 540 DK-8000 Aarhus C Denmark. (email: yeo@daimi.au.dk). The third author wishes to acknowledge financial support from the Danish Research Council (under grant 9800435). He also wishes to acknowledge the hospitality of Department of Mathematics and Statistics, University of Victoria, where part of this research was carried out.