Institut for Matematik og Datalogi
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JFB

Introduction to Information Technology
E01 – Note 5

Lecture, October 1

We continued talking about algorithms from chapters 2 (from section 2.3.2 to the end of the chapter) in the textbook. In addition, Maple, a program for symbolic computation, was presented and demonstrated, as were statistics in Excel.

Lecture, November 2

We will continue in chapter 3 of the textbook. In addition, some types in Maple (for example, integers, reals, sets, lists, lists of lists, and functions) will be demonstrated and discussed. See the lab description for the relevant reading in *Calculus the Maple Way*.

Lecture, November 16

We will finish chapter 3 and begin on section 14.7 of chapter 14. Spreadsheets and statistics in Maple will be demonstrated.

Announcement

The printer in the computer laboratory can now be used. There are directions on the course's Web page describing how to connect to it. Each student in the natural sciences can print a total of 250 pages each semester free of charge. It should be possible to pay for extra pages if necessary.

Primary Lab 5 - for week 45

Read the following from *Calculus the Maple Way* before coming to the lab: the sections "Maple Does Derivatives" from Lab 4, "Higher-Order Derivatives" and "Antiderivatives" from Lab 5, "Extreme Values" from Lab 6, "Integrals" from Lab 11, and "Taylor Series" from Lab 16. Bring *Calculus the Maple Way* to your lab.

The purpose of this lab is to introduce you to more capabilities of Maple, with emphasis on those useful in MAT A.

Exercise 1

Start by opening the Maple 7 program. Now try some differentiation:
To compute the derivative $\frac{d}{dx}(x^3 + 6x - 7)$, type `diff(x^3+6*x-7,x);`. The second argument x indicates that the derivative is with respect to x; the same function can be used for partial derivatives and higher order derivatives. For example, try `diff(x^3+6*x-7,x,x);` or `diff(x^3+6*x-7,x$2);`, both of which should give you the second derivative with respect to x.

If you do not do Exercise 6, do the following: Do all four parts of problem 1 on page 35 of *Calculus the Maple Way*. (Simplify when helpful.) Then, do problem 7 (note that composition of functions was explained on page 17).

Exercise 2

To compute the integral of $\frac{1}{x^4} - \frac{1}{x^3}$, type `int(1/x^4 -1/x^3,x);`. (You can also do this with the expression palette. Try that, too.) Try right clicking on the result and choosing **Simplify** to see if you like that form better. On the result of this, you can right click and choose **Differentiate** to check your result. This result looks a bit different from what you started with, so right click on it and choose **Expand**. Here is a case where *expand* simplifies!

If you do not do Exercise 6, do the following: Compute the integrals of the functions in parts (b) and (d) of problem 1 on page 35 of *Calculus the Maple Way*. Compute the following definite integrals of the these functions: $\int_{-2}^{2} x(x-1)(x+2)\, dx$ and $\int_{-2}^{2} \sin(x) + \cos(3x)\, dx$.

Exercise 3

Finding the minima and maxima of $\frac{\cos(x)}{1+x^2}$.
This exercise shows some other methods than those emphasized in *Calculus the Maple Way*, and it demonstrates how to overcome some common problems with Maple.

Type `formula := (cos(x))/(1+x^2);`, and then right click on the result to plot the function. Now differentiate the function, assign the result to a variable A, and plot the derivative. The minimum and maximum will occur where the derivative is zero, so solve for that. The result just says that one solution of that equation is also a root of the given function. Typing `evalf(%);` will give you the value of that root, which should be zero. Now you can check what value the original function had at zero with `eval(formula,x=0)`, which evaluates the formula you assigned to the variable *formula* at $x = 0$. You can see from your plot that this is the maximum.

To specify that the next calculation (to find the minima) should have more precision, say 30 digits precision, you type `Digits:=30;`. Now, you can try to find another root of the derivative. You can specify that you want to avoid the root at zero by typing `fsolve({A=0},{x},avoid={x=0});`. Check the plot to see if this looks like a minimum. Then, evaluate your formula at this point. Does the result look correct? To find the root between 1 and 3, you can again use *fsolve*: `fsolve(A=0,x,1..3);`. If you try to find a root between 1 and infinity, it will find still another root. Try it. To specify infinity, you can choose it from the symbol palette, or simply type `infinity`.

If you do not do Exercise 6, do the following: Do problem 8 on page 51 of *Calculus the Maple Way*.

Exercise 4

Taylor series approximations.
To find Taylor series approximations of functions, you use the function *taylor*. Try typing `taylor(f(x),x=a);` to find out what sort of result you will get. The result should be symbolic if you have not already defined f, and the D's represent derivatives. Notice that you have specified that the expansion should be with respect to the variable x about the point a, and it automatically goes up to order 6.

Now define a specific function $f(x) = e^{-x^2}$ with `f:=x->exp(-x^2);` and compute the Taylor series expansion around $x = 0$ as `t:=taylor(f(x),x=0);`. The result is not a polynomial since it contains the order of magnitude term at the end. Change it to a polynomial using `poly6 := convert(t,polynom);`, and it should drop the term $O(x^6)$. Now you can see how good this approximation is by plotting it and your function f together: `plot([f(x),poly6],x=-2..2);`.

You can specify that the approximation should be to a higher order by giving a third argument to the *taylor* function. For example `t15:=taylor(f(x),x=0,15);`. Convert this to a polynomial called *poly15*. Now you should plot $f(x)$, *poly6* and and *poly15* together to compare them, again with x between -2 and 2; use the colors *red, blue*, and *plum*. (When plotting a list of functions, you can specify the colors of the curves, using a argument **color** to **plot**. If plum is too faint, try orange.) Notice that the higher order approximation makes a difference.

Exercise 5

Descriptive statistics. Type `with(stats);` to activate that package and see the subpackages available with the statistics package. Then type `with(describe);` to see the subpackages and functions available there. Create a list with 20 numbers and assign it to a variable Z. For example, `Z:=[59,20,94,85,49,66,80,49,58,29,74,77,94,39,89,56,72,64,64,59];`. Now you can find the mean, median, and standard deviation, by using those three functions with the argument Z. For example, `standarddeviation(Z);`. If you gave integers as input, two of the three results should be approximated. Right click on those values and approximate to 5 digits precision.

Exercise 6

Optional. If you do this, you may skip all the problems from *Calculus the Maple Way*.

Write your own Maple function to compute the sum of the squares of the values in a list (for example, those in the list Z from Exercise 5). If your list L has n values, then you should compute $\sum_{i=1}^{n}(L[i])^2$. Do it without using the Maple function **sum**. It is possible in Maple to define a function in terms itself. Thus, you could define two parameters for your function, the list, plus the length of the list. When the length is zero, your sum should be zero. Can you define your function for a list of length n in terms of a result of applying your funciton to a list of length $n-1$? You will need the **if** statement in Maple, which you can read about from **Help**.

Check that your function works on some examples.

Exercise 7

Save your worksheet. E-mail your worksheet to your lab instructor as an attachment. Remember to logoff your computer, but do not push any of the buttons on it.