DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE
ODENSE UNIVERSITY

COMPUTER SCIENCE COLLOQUIUM

Extending Matchings in Graphs: a survey

Michael D. Plummer
Department of Mathematics
Vanderbilt University

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Abstract: Let G be a graph on p vertices and let k be an integer such that $1 \leq k < p/2$. A graph G is k-**extendable** if every matching of size k in G is a subset of a perfect matching in G. In 1979, Sumner asked: what graphs have the property that every matching extends to, i.e., is a subset of, a perfect matching? He called such graphs **randomly matchable**. The answer was simple: only the even complete graphs K_{2n} and the complete bipartite graphs $K_{n,n}$. Somewhat later, Lesk, Plummer and Pulleyblank investigated the structure of the more general class of graphs in which every matching extends to a *maximum* matching. Their characterization was rather complicated, but it did lead to a polynomial recognition algorithm for this class.

The study of k-extendable graphs arises naturally, not only as a generalization of Sumner's randomly matchable idea, but also in the decomposition theory of graphs with respect to their maximum matchings. Central to this theory is the idea of a *bicritical* graph. A graph G is said to be **bicritical** if $G-u-v$ has a perfect matching for all choices of vertices $u, v \in V(G), u \neq v$. Bicritical graphs play a central role in the decomposition theory mentioned above and their structure is far from completely understood. It was shown in 1980 that every k-extendable graph is also $(k-1)$-extendable and that any 2-extendable graph is either bipartite or bicritical. Thus for *non-bipartite* graphs, if $\mathcal{G}_k$ denotes the class of those which are k-extendable and $\mathcal{G}_b$ those which are bicritical, we have a nested sequence of graph classes: $\mathcal{G}_1 \supset \mathcal{G}_b \supset \mathcal{G}_2 \supset \mathcal{G}_3 \supset \cdots$

Moreover, it is known that each of the containments is proper.

We will attempt to survey the considerable amount of recent work that has been done in the area of matching extension.

Bjarne Toft