Institut for Matematik og Datalogi
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JFB

Cryptology – F05 – Lecture 5

Lecture, February 25

We finished with section 2.3. We covered all but section 3.6 (which we began on) in chapter 3 in the textbook, skipping much of the first four sections. The original specification (which can be found through the course's homepage) will be used as the basis for the description of AES.

Lecture, March 4

We will finish section 3.6 and begin on chapter 5 in the textbook. Note that section 5.2.1 was covered earlier.

Lecture, March 11

We will continue with chapter 5.

Problem session March 9

Note that section numbers referred to in these problems are in the Rijndahl specification, which you can find on the Web. For problems using Maple, it is fine if you use Mathematica instead.

1. In the original description of Rijndael, it says that x^4+1 (which is used to create the matrix for the MixColumn operation) is not irreducible over $GF(2^8)$. What are its factors? Try the function `Factor` in Maple, using `mod 2`. Check that the `mod 2` makes a difference by also trying to factor it with `factor`.

   Check that $x^8+x^4+x^3+x+1$ is irreducible over $GF(2)$. Check the multiplication done in the example in section 2.1.2 using the `modpol` function in Maple.

Find the inverse of $x^7 + x^5 + x^3 + 1$ modulo $x^8 + x^4 + x^3 + x + 1$. Try the function `powmod` using the exponent -1. Check that your answer is correct using `modpol`.

2. Why do you think $x^4 + 1$ was used, rather than an irreducible polynomial? Why are there no problems that it is not irreducible?

3. Check that the definition given for the polynomial $d(x)$ in section 2.2 is correct. In Maple, I found it useful to multiply the polynomials, use the right mouse button to find `collect` and `x`, and repeatedly add on appropriate multiples of $x^4 + 1$. There might be a better way, but I couldn't get the `modpol` function to do anything in this case.

   Similarly, check that the polyomial $d(x)$ used in MixColumn in section 4.2.3 is correct.

   This problem is probably just about as easy to do by hand.

4. Find the inverse transformation for ByteSub in section 4.2.1. To find the inverse modulo 2 of the matrix, you can use the `Inverse` function in Maple. To create the matrix, you can use the function `Matrix` (in the LinearAlgebra package, so you have to type `with(LinearAlgebra);` first) and list the matrix row by row. For example, to create the matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, you can type `A:=Matrix([[1,2],[3,4]]);`. To check the result, you can multiply two matrices, A and B using `C:=A.B;`. To reduce all the elements of the matrix modulo 2, you can use the `Map` function, for example as `Map(modp,C,2);`.

5. Look at problems 5.3, 5.6, and 5.7 in the textbook. If you are at all unsure of how to do them, please do them. Even if you are not unsure, you might consider this an opportunity to try using Maple. The following Maple functions should be useful: `igcdex` (extended Euclidean algorithm for integers), `mod` (where the operation `&^` should be used for more efficient modular exponentiation - try them both to compare), `msolve` (solve equations in $\mathbb{Z}_m$), and `chrem` (Chinese Remainder Algorithm).

6. Another easy problem. Let $n = 143$ be a modulus for use in RSA. Choose a public encryption exponent e and a private decryption ex-

ponent d which can be used with this modulus. Try encrypting and decrypting some value to see that the exponents you have chosen work.

7. Suppose you as a cryptanalyst intercept the ciphertext $C = 10$ which was encrypted using RSA with public key $(n = 35, e = 5)$. What is the plaintext M? How can you calculate it?

8. In an RSA system, the public key of a given user is $(n = 3599, e = 31)$. What is this user's private key?