Institut for Matematik og Datalogi
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March 18, 2002
JFB and KM

Discrete Mathematics with Applications
F02 – Lecture 6

Lecture, March 4

We finished section 2.5, along with Lamé's Theorem from chapter 3.

Lecture, March 11

Klaus Meer will begin on chapter 4 which deals with techniques for counting. We'll introduce permutations, r-permutations and r-combinations and count their number. The binomial coefficients are defined. They play an important role in performing the above mentioned counting tasks.

Lecture, March 18

We continue the study of counting methods and deal with the binomial theorem (section 4.3), permutation and combinations with repetition (section 4.6), the Pigeonhole principle (section 4.2) and permutations of sets with indistinguishable objects (section 4.6). Then we turn to the principle of inclusion and exclusion (section 5.5).

Problems to be discussed on March 19

- Problems 40, 42, 46, 52 from section 4.1.

- Problems 28, 32, 60 from section 4.3.

Maple problems

Maple has an own package for dealing with counting issues. In order to use it start your program with the commands

```
> restart;
> with(combinat);
```

You will see on the screen a list of available functions within the combinat package. Read in the help menu the descriptions of the functions numbperm,permute, numbcomb and choose. Try some examples by yourself; in particular, include examples where indistinguishable objects occur.

The function `binomial(n,k)` is available as standard. It computes the binomial coefficient *nchooser*. Use it to generate the entries in the first 10 rows in Pascal's triangle (you can use the command seq here).

Next, compute the polynomial $p(x) := (1 + x)^n$ for some values of n (use the `expand` command). Chose a number $k \leq n$ and compute the coefficient of x^k in p. This can be done using the command `coeff(p, x^k)`. Finally, compare it with the binomial coefficient $\binom{n}{k}$.

<u>**Problem:**</u> This is a more involved Maple problem. We want to use Maple in order to get an idea about a special property of Pascal's triangle. We shall formulate the corresponding precisely (and give a rigorous proof of it) in one of the next exercises.

i) Use Maple to compute for all integers $n \leq 50$ the number of odd binomial coefficients in row n, i.e. the number of odd $\binom{n}{k}, 0 \leq k \leq n$.

   You can use a function `mod 2` which computes modulo 2.

   Can you say something about the number of odd entries with respect to n? Do you realize some regularity?

ii) For a number n as above, use Maple to compute its binary representation (with the function `convert(n, base, 2)`).

   Now, count the number of 1's in that binary representation. Maple has a function `nops(NAME)`, which gives the number of elements in a list NAME. You can use it in order to determine the length of a binary representation (the command convert gives a list as result). Finally, compute 2 to the power of number of 1s you found. Do that again for, say, the first 50 values of n. Compare with i).

Second required assignment

Rules are the same as for the first assignment. Solutions must be turned in by Monday, March 25 at 10:15h. Solutions handed in later will not be accepted.

- Solve problem 24 from section 2.5. Show how you got your answers.
- With RSA there are often recommendations to make the public exponent $e = 3$. In this case, one must have $p \equiv q \equiv 2 \pmod 3$. Prove this, i.e. prove that neither p nor q can be congruent to 0 or 1 modulo 3 if $e = 3$.
- Solve problem 24 from section 4.3. We assume the alphabet to contain 26 letters. Give your answers by writing down correct “terms“ rather than concrete numbers (for example, the expression 26^5 is better than the evaluated answer 11881376). In any case, you have to explain how the result was obtained. Here, using Maple is not accepted.
- Solve problem 54 from section 4.3.