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JFB and KM

Discrete Mathematics with Applications
F02 – Lecture 7

Lecture, March 11

Klaus Meer began teaching on chapter 4 which deals with techniques for counting. We introduced permutations, r-permutations and r-combinations and counted their number. The binomial coefficients were defined and the binomial theorem was proven. We also studied combinations with repetitions.

Lecture, March 18

We continue to study counting techniques by analyzing the Pigeonhole principle (section 4.2) and permutations of sets with indistinguishable objects (section 4.6). Then we turn to the principle of inclusion and exclusion (section 5.5).

Lecture, March 25

The next technique we study is the solution of linear recurrence relations. We introduce the Fibonacci numbers as solution of a recurrence relation of degree two. We shall mainly concentrate on solving such degree two recurrence relations (sections 5.1 and 5.2).

Problems to be discussed on March 26

- Problems 18, 20, (35) from section 4.6

- Problems (16), 38 from section 5.1

- Problems 4, 8, 16, (18), (19), 22, (23) from section 5.6
- In this problem we take up again the Maple problem on Lecture Note 6. If you performed well, then your guess about the number of odd entries in row n of Pascal's triangle resulted in the following

  <u>**Theorem:**</u> For any $n \in \mathbb{N}$ the number of odd entries in Pascal's triangle is 2 raised to the power one ones in the binary expansion of n.

  We want to prove this result. Towards this end, first try to prove

  $$(1+x)^{2^i} \mod 2 = 1 + x^{2^i} \ \forall i \in \mathbb{N} .$$

  Here, taking a polynomial $(1+x)^{2^i}$ modulo 2 means to take its coefficients modulo 2. The above statement says that all coefficients of $(1+x)^{2^i}$ except the first and the last are even.

  Next, use this result for computing the number of odd coefficients in $(1+x)^n$ for an arbitrary n by considering its binary expansion

  $$n = \sum_{i=0}^{s} b_i \cdot 2^i .$$

Maple problems

Use Maple to compute the number of strings of length 9 with components in $\{0,1\}$ such that there occur both a substring with at least three consecutive 0s and a substring with at least three consecutive 1s. Your result should be 160.