Institut for Matematik og Datalogi
Syddansk Universitet

December 8, 2004
JFB

DM19 – Algorithms and Complexity – E04 – Lecture 14

Announcement 1

Husk kursustilmelding senest fredag, 10/12 2004. Man skal tilmelde sig alt via fakultetets tilmelding. Hvis man tager valgfri kurser eller har søgt instruktorat på IMADA, skal man yderligere oplyse om ens samlede kursusvalg via IMADA's hjemmeside. Dette er af hensyn til skemalægningen af valgfri kurser.

Announcement 2 – repeat

The sign-up sheets for the oral exam are in IMADA's departmental office now. Please sign up in good time. The dates of the exam are January 3, 4, and 5. The questions can be found on the course's home page.

Lecture, December 6

We finished with branch-and-bound from the notes and began on heuristics (also from the notes), covering up through and including section 10.4.2 on genetic algorithms.

Lecture, December 13

We will finish with heuristics from the notes and begin on on-line algorithms, also from the notes.

Lecture, December 20

We will finish with on-line algorithms.

Problems to be discussed December 21 (10:15 in U49D) and December 17

1. Design a genetic algorithm for the MAX-SAT problem.

2. Show that LFU is not competitive for any $k \geq 3$. This means that for any function $f(k)$ and any constant a, there is a request sequence on which LFU has C_{LFU} faults, while an optimal off-line algorithm has C_{OPT} faults, and $CLFU > f(k) \cdot C_{OPT} + a$. LFU is defined as follows: For each page in the system, LFU keeps a counter keeping track of how many times the page is requested. When it is necessary to evict a page, the page chosen has the smallest counter value. (If this smallest value is not unique, any of the pages with that smallest value may be chosen.)

3. Prove Thm 1 (on page 3 of the notes on on-line algorithms) for FIFO. (The proof given in the notes is only for LRU.)

   Note: The next 2 problems are from the textbook *Online Computation and Competitive Analysis*, by Allan Borodin and Ran El-Yaniv.

4. If a deterministic algorithm for paging is c-competitive, then $c \geq k$, even if there are only $k+1$ pages in the system, where the fast memory can contain k pages. Prove that MARKING is H_k-competitive, when there are only $k+1$ pages in the system.

5. Show that in general, MARKING is not H_k-competitive. (Hint: It is sufficient to consider the case $k = 2$, with $N = 4$ pages total.)

6. Consider the on-line bin-packing problem: There is a large supply of bins, all of capacity 1. The request sequence is a sequence of objects $< r_2, ..., r_m >$. Object r_i has size s_i, which is known when the object arrives. The goal is put each object in some bin, so that no bin contains objects whose sizes add to more than 1, using as few bins as possible. The algorithm First-Fit places an object in the first bin in which it fits. Show that First-Fit is 2-competitive. (Actually, it is 17/10-competitive, but that's much harder to show.)

7. Show that the First-Fit algorithm for bin-packing cannot be better than 5/3-competitive. (Hint: use items of size $\frac{1}{7} + \epsilon$, $\frac{1}{3} + \epsilon$, and $\frac{1}{2} + \epsilon$, where ϵ is very small.)