Institut for Matematik og Datalogi
Syddansk Universitet

December 8, 2005
JFB

DM19 – Algorithms and Complexity
E05 – Lecture 14

Lecture, December 7

We finished with branch-and-bound from the notes and began on heuristics (also from the notes), covering up through and including section 10.4.1 on simiulated annealing.

Lecture, December 14

We will finish with heuristics from the notes and begin on on-line algorithms, also from the notes.

Lecture, December 21

We will finish with on-line algorithms.

Problems to be discussed in week 51

1. Design a genetic algorithm for the MAX-SAT problem.

2. Show that LFU is not competitive for any $k \geq 3$. This means that for any function $f(k)$ and any constant a, there is a request sequence on which LFU has C_{LFU} faults, while an optimal off-line algorithm has C_{OPT} faults, and $CLFU > f(k) \cdot C_{OPT} + a$. LFU is defined as follows: For each page in the system, LFU keeps a counter keeping track of how many times the page is requested. When it is necessary to evict a page, the page chosen has the smallest counter value. (If this smallest value is not unique, any of the pages with that smallest value may be chosen.)

3. Prove Thm 1 (on page 3 of the notes on on-line algorithms) for FIFO. (The proof given in the notes is only for LRU.)

4. Consider the algorithm Flush-When-Full (FWF) for paging. When it faults on a page p, if there are less than k pages in cache, it simply brings p into cache. If there are k pages in cache, FWF "flushes" cache, removing all pages (it doesn't have to actually remove them; it can just forget that they are there and set its counter for the number of pages in cache to zero), and then brings p into cache. Prove that FWF has competitive ratio k.

Note: The next 2 problems are from the textbook *Online Computation and Competitive Analysis*, by Allan Borodin and Ran El-Yaniv.

5. If a deterministic algorithm for paging is c-competitive, then $c \geq k$, even if there are only $k+1$ pages in the system, where the fast memory can contain k pages. Prove that MARKING is H_k-competitive, when there are only $k+1$ pages in the system.

6. Show that in general, MARKING is not H_k-competitive. (Hint: It is sufficient to consider the case $k = 2$, with $N = 4$ pages total.)

7. Consider the bin-packing problem from problem 35-1 in the textbook. Note that the First-Fit algorithm described there is on-line. Show that the First-Fit algorithm for bin-packing cannot be better than 5/3-competitive. (Hint: use items of size $\frac{1}{7} + \epsilon$, $\frac{1}{3} + \epsilon$, and $\frac{1}{2} + \epsilon$, where ϵ is very small.)