Institut for Matematik og Datalogi
Syddansk Universitet

March 5, 2007
JFB

DM508 – Algorithms and Complexity – F07
Lecture 9

Lecture, February 22

We continued with NP-Completeness, covering Cook's Theorem and the proof that 3-SAT is NP-Complete.

Lecture, February 27

We will continue with NP-Completeness, doing a number of reductions.

Lecture, March 1

We will finish NP-Completeness and begin on approximation algorithms, covering the introduction to chapter 35 and section 35.2.1.

Problems to be discussed on March 2

1. 34.5-5 (Warning: it is tempting to think that this one is completely trivial; it is not. Also, to make this easier, you may redefine the Set Partition problem to allow the same value appearing more than once), 34.5-6.
2. 34-2, 34-3.
3. 34-1a, 34-1b, 34-1c.
4. 34-4a, 34-4b, 34-4c.

Assignment due Friday, March 9, 12:15

Note that this is part of your exam project, so it must be approved in order for you to take the exam in March. This means that the work must be your group's own work. No part of your work may be taken from another source, and you may not work with others not in your group. You may work in groups of two or three. You may write your solutions in English or Danish, but write very neatly if you do it by hand.

1. In the *half 4-CNF satisfiability* problem, a 4-CNF formula (CNF form, with exactly 4 literals per clause) F is given. One knows that at least half of the clauses are satisfiable by any truth assignment. The problem is to determine if there exists a truth assignment to the variables of F which satisfies the entire formula. Prove that the half 4-CNF satisfiability problem is NP-complete.

2. In the *Partition* problem, a finite set A is given, along with a positive integer size $s(a)$, for each $a \in A$. The problem is to determine if there exists a subset $A' \subset A$ such that $\sum_{a \in A'} s(a) = \sum_{a \in A \setminus A'} s(a)$ (i.e. can you partition the set into two subsets so the sizes of the items add together to exactly the same amount?).

   In the *Bin Packing* problem, a positive integer bin capacity B, a positive integer K, and a finite set A is given, along with a positive integer size $s(a)$, for each $a \in A$. The problem is to determine if there exists a partition of A into disjoint sets $A_1, A_2, ..., A_K$ such that the sum of the sizes of the items in each A_i is B or less. (This partition into disjoint sets gives a packing into K bins.)

   - Partition is known to be NP-complete. Using this fact, prove that Bin Packing is also NP-complete.
   - Show that if there is an algorithm for Bin Packing which runs in time $f(n)$ for some function f, then there is an algorithm for the cost version of Bin Packing (find the cost of the packing which uses fewest bins, where the cost is the number of bins used) which runs in time $O(p(f(n))$ for some polynomial p.
   - Suppose that you know that in the optimal packing there are at most 10 items per bin and that there is an algorithm for Bin Packing which runs in time $f(n)$ for some function f. Show that then there is an algorithm for finding an optimal packing (one that used the smallest possible number of bins) which runs in time $O(p(f(n))$ for some polynomial p.