Institut for Matematik og Datalogi
Syddansk Universitet

February 18, 2009
JFB

# DM508 – Algorithms and Complexity – 2009
# Lecture 5

## Lecture, February 16

Cancelled due to illness.

## Lecture, February 18

We continued with NP-Completeness, covering Cook's Theorem and reductions showing that 3-SAT and CLIQUE are NP-Complete.

## Lecture, February 23

We will continue with NP-Completeness, covering more reductions from known NP-Complete problems.

## Exercise session for February 19 cancelled

Those problems will be discussed on February 24.

## Problems to be discussed on February 26

Do problems:

1. 34.5-5 (Warning: it is tempting to think that this one is completely trivial; it is not. Also, to make this easier, you may redefine the Set Partition problem to allow the same value appearing more than once), 34.5-6.
2. 34-2, 34-3.
3. 34-1a, 34-1b, 34-1c.
4. 34-4a, 34-4b, 34-4c.
5. 17.1-2, 17.1-3 (if we have begun to cover amortized analysis)

## Assignment due Monday, March 2, 12:15

Note that this is part of your exam project, so it must be approved in order for you to take the exam in April, and you may not work with or get help from others not in your group. You may work in groups of two or three. You may write your solutions in English or Danish, but write very neatly if you do it by hand.

1. Prove that the following problem, which we will call Matrix–Processor Assignment, is NP-Complete. Given an $n \times m$ matrix (array), divided into rectangles (for example, all elements both in some row between 13 through 63, inclusive, and in some column between 24 and 37, inclusive, consitute a rectangle with $51 \cdot 14 = 714$ elements and area 714), and a set of $k$ processors, $P_1, P_2, ..., P_k$, with capacities, $c_1, c_2, ..., c_k$, is there an assignment of rectangles to processors, such that for each processor, $P_i$, the total area of the rectangles assigned to it is at most $5c_i$? (The idea is that we want all the processing of the rectangles completed by a certain time, and their processing time is proportional to their area. Some processors are more powerful than others.) The values $n, m, k$ can vary from one instance of the problem to another, and the rectangles cover the entire matrix and do not overlap. This division into rectangles is given as part of the input to the problem, as are the capacities of the processors.

   You may assume that Bin-Packing is NP-Complete. In Bin-Packing one is given a set of $n'$ items with positive integer sizes $s_1, s_2, ..., s_{n'}$. The question is, can you pack all of the items in at most $L$ bins, such that no bin gets packed with items with total size more than $B$?

2. The Matrix–Processor Assignment problem stated above is the recognition version of the problem (a decision problem). An evaluation version of this problem is to find the minimum number of the given processors which need to be used to assign all rectangles to some processor (satisfying the constraint that the total area assigned to processor $P_i$ is at most $5c_i$). Suppose you are given a black-box, polynomial time algorithm for solving the Matrix–Processor Assignment problem. How would you use it to get a polynomial time algorithm for this evaluation version of the problem? Present the algorithm and argue that it is polynomial time.

3. Prove that the following problem is NP-Complete: Given a collection $C$ of finite sets and a positive integer $K \leq |C|$, does $C$ contain $K$ disjoint sets?