Institut for Matematik og Datalogi
Syddansk Universitet

February 1, 2010
JFB

DM508 – Algorithms and Complexity – 2010
Lecture 2

Lecture, February 1

We began with an introduction to the course. Lower bounds from section 2.4 of the first part of the notes were discussed (part of this is also in section 8.1 of the textbook). We also covered section 3.1, and the algorithm, but not the lower bound, of section 3.2 of those notes.

Lecture, February 2

We will finish sections 3.2, 3.3 and 3.5 of the DM508 notes, plus median finding from chapter 9 (sections 9.2 and 9.3) in the textbook. We may also begin on NP-completeness, from chapter 34 in the textbook and the section by Papadimitriou and Steiglitz from the course notes.

Lecture, February 8

We will begin on NP-completeness, from chapter 34 in the textbook and the section by Papadimitriou and Steiglitz from the course notes. Note that I will use mostly use the notes for Cook's Theorem (the proof that SAT is NP-Complete).

Problems to be discussed on February 9

1. Consider a company with a customer service department consisting of an automatic system and two representatives. The company has customers in three cities, Detroit, Chicago, and San Francisco. A request for service requires one of the service representatives to be in that city to service it. Each city has an apartment for the service representative, who will stay there until called to another city. Assume the flight between Chicago and Detroit costs only a fraction $1/d$ of what the flight between Chicago and San Francisco costs, and that it is impossible to fly from Detroit to San Francisco without changing planes in Chicago and paying the sum of the ticket costs from Detroit to Chicago and from Chicago to San Francisco. (Assume that all costs are symmetric, so that it costs exactly as much fly the opposite direction between two cities.) The company wishes to minimize the amount it spends on plane tickets. Assume in what follows that there is initially one service representative in

Detroit and one in San Francisco. In the following, assume that if the next request for service is in a city where there is already a service representative, then no service representative flies anywhere, and this costs nothing.

(a) Show that in the worst case, any algorithm has cost at least $n \cdot f$ on a sequence of requests of length n, where f is the cost of the cheaper flights.

(b) Consider the Greedy algorithm which always requires the closer representative (note that the closer pair of cities has the lower cost airfare) to travel (when travel is necessary). Also consider the algorithm Dummy which, on requests in Chicago always sends the representative which is in San Francisco (when travel is necessary), but otherwise sends the closer representative. Show that for any constant c, there exists a sequences where Dummy pays a factor more than c times what Greedy pays and another sequence where Greedy pays a factor more than c what Dummy pays.

Do problems:

1. 9.3.1, 9.3.2, 9.3-3, 9.3.4, 9.3-7, 9.3-9,
2. 34.1-3, 34.1-5, 34.2-3.
3. Suppose that there is a language L for which there is an algorithm that accepts any string $x \in L$ in polynomial time and rejects any $x \notin L$, but this algorithm runs in super-polynomial (more than polynomial) time if $x \notin L$. Argue that L can be decided in polynomial time.
4. Define an algorithm to show that SATISFIABILITY is in NP.