

DM508 – Algorithms and Complexity – 2011 Lecture 6

Lecture, February 14

We continued with NP-Completeness, covering more reductions from known NP-Complete problems, proving 3-SAT, CLIQUE, VERTEX COVER, INDEPENDENT SET, and HAMILTONIAN CIRCUIT NP-Complete.

Lecture, February 21

We will finish NP-Completeness (finishing a reduction from 3-SAT to HAMILTONIAN CIRCUIT and covering SUBSET-SUM) and begin on amortized analysis from chapter 17 of the textbook.

Lecture, February 24

We will cover Fibonacci heaps from chapter 19 in the textbook.

Problems to be discussed on March 2

Do problems: 17.1-2, 17.1-3, 17.2-3, 17.3-2, 17.3-7, 17-3,

Assignment due Monday, February 28, 8:00

Note that this is part of your exam project, so it must be approved in order for you to take the exam in April, and you may not work with or get help from others not in your group (though you may talk with an “instruktor” or myself). You may work in groups of two or three. You may write your solutions in English or Danish, but write very neatly if you do it by hand. Submit the assignment via Blackboard as one PDF file.

Suppose G is a graph representing an adventure game, “World of Lightcraft”. Each vertex in G represents a “room”, and each edge represents a way of going from one room to another (i.e., a magic tunnel). Each magic tunnel has a monster which will kill you if it is dark and you have not put the monster to sleep. Originally, all of the magic tunnels are dark. The goal is to light all of the magic tunnels and have them all lit simultaneously. A magic tunnel can be lit by turning on the a special light in one of the rooms it connects

(turning on a light in a room lights all of the magic tunnels going off from it). To turn on a light, you need a special card which must be left in the base of the light as long as the light is turned on. You start in a room, called the “Entrance Hall”, You have a supply of s gas capsules, and each one can put one monster to sleep long enough for you to get through its magic tunnel. The “Room of Magic” contains one special wand which will put any monster you meet to sleep instantly. It also contains t special cards for lamps. The “Entrance Hall” is a different room from the “Room of Magic”. You have a map of G in advance. The question is, “Is it possible to win this game by lighting all tunnels simultaneously?”

1. Show that it is NP-Complete to determine if an instance of World of Lightcraft is winnable. (An instance consists of G , s , and t .) This is the recognition version of the problem.
2. Suppose that you have an efficient (polynomial time) algorithm A for determining if an instance of World of Lightcraft is winnable (a black-box polynomial time algorithm for the recognition version). Give a polynomial time algorithm (using A) to determine an efficient strategy for winning the instance when it is winnable. (Thus, you should solve the optimization version, using a black-box, polynomial time algorithm for the recognition version of the problem.)
3. Suppose each room has at most two magic tunnels going off from it. Is the recognition version of the problem still NP-Complete? Prove your answer.