Institut for Matematik og Datalogi
Syddansk Universitet

February 15, 2012
JFB

DM508 – Algorithms and Complexity – 2012
Lecture 4

Lecture, February 6

We covered the lower bound on median finding from section 3.5 in the DM508 notes. We began on NP-completeness, from chapter 34 in the textbook and the section by Papadimitriou and Steiglitz from the course notes, defining P and NP and reductions.

Lecture, February 10

We will continue with NP-Completeness, covering more reductions from known NP-Complete problems. We may begin on Cook's Theorem, proving that SATISFIABILITY from the section by Papadimitriou and Steiglitz from the course notes.

Lecture, February 13

We will cover Cook's Theorem, proving that SATISFIABILITY from the section by Papadimitriou and Steiglitz from the course notes.

Problems to be discussed on February 15

Do problems:

1. 34.5-4. (you may check on pages 1228–1129 for a hint, which is 1044–1045 in the second edition).
2. 34.5-5 (Warning: it is tempting to think that this one is completely trivial; it is not. Also, to make this easier, you may redefine the Set Partition problem to allow the same value appearing more than once), 34.5-6.
3. 34-2, 34-3.
4. 34-1a, 34-1b, 34-1c.
5. 34-4a, 34-4b, 34-4c.

Assignment due Wednesday, February 22, 10:15

Note that this is part of your exam project, so it must be approved in order for you to take the exam in March, and you may not work with or get help from others not in your group (though you may talk with Magnus Find or myself). You may work in groups of two or three. You may write your solutions in English or Danish, but write very neatly if you do it by hand. Submit the assignment via Blackboard's "SDU Assignment" as one PDF file. Note that you should not turn in a paper copy. Turn in one assignment per group.

Consider a game where the board used is a map showing how to connect a number of cities with railroads. A player is given a fixed number, N, of railway pieces at the start of the game, along with some number, m, of tasks. A task is the name of two cities, the specification of what route should be used between them (on the board), and the number of railway pieces needed to connect them. There may be overlap, so that the same railway piece can be used for more than one different task, but only the given most direct route between the two cities listed in a task may be used. The object is to complete some whole number of tasks, using as many of the N railway pieces as possible: The players score is equal to the number of pieces used correctly. The problem the player needs to solve is to choose a subset of the m tasks to maximize the number of railway pieces used.

An obvious decision problem to consider here is the following: Given a board, a value N, and a set of m tasks, is it possible for the player to use all N railway pieces?

1. Show that this decision problem is NP-complete.
2. Show that if your decision problem can be solved in polynomial time, that the problem of finding the optimal set of tasks can also be solved in polynomial time.
3. Show that if there is only one fixed board, that finding the optimal solution for a given set of tasks is no longer NP-hard (unless P=NP).