Institut for Matematik og Datalogi
Syddansk Universitet

September 27, 2018
JFB

DM551 – Algorithms and Probability – 2018
Lecture 8

Lecture, September 26

We finished section 13.1 and up through theorem 13.5 in section 13.2 in *Algorithm Design* by Kleinberg and Tardos.

Lecture, October 1

We will finish section 13.2 and cover sections 13.3 and 13.4 in *Algorithm Design* by Kleinberg and Tardos.

Lecture, October 8

We will cover section 13.5 in Kleinberg and Tardos. We will also analyze the expected number of comparisons done by Randomized Quicksort, using section 7.4.2 of *Introduction to Algorithms*, 3rd edition, by Cormen, Leiserson, Rivest, and Stein (CLRS). The next topics will be from sections 13.9–13.10 in Kleinberg and Tardos.

Problems to be discussed on October 10

1. Exercises on pages 782–793: 6 and 7a.

2. Discuss Solved exercise 2 on page 776 of Kleinbert and Tardos.

3. This problem concerns finding a spanning bipartite subgraph, instead of a spanning tree. A *bipartite graph* $G = (V, E)$ is such that $V = V_1 \cup V_2$ and $V_1 \cap V_2 = \emptyset$ ((V_1, V_2) is a partition of V), and all edges is E have one endpoint in V_1 and one endpoint in V_2. (Any tree is a bipartite graph, but any graph with a cycle of odd length is not.) For an arbitrary graph $G = (V, E)$ (not necessarily bipartite), a *spanning bipartite subgraph* G' of G is defined by a partition (V_1, V_2) of V, and the edges of G', E', are the edges in E with one endpoint in V_1 and one endpoint in V_2.

   Consider the following randomized algorithm which finds a spanning bipartite subgraph of an arbitrary graph $G = (V, E)$: Independenlty for each vertex $v \in V$, Rand-Alg randomly decides if vertex v is in V_1 or V_2, choosing V_1 with probability 1/2. It sets $E' = \{e \in E \mid e \text{ has one endpoint in } V_1 \text{ and one enpoint in } V_2\}$.

(a) Give a lower bound for the expected number of edges in E' (in terms of $|E|$), when Rand-Alg is run on a graph $G = (V, E)$.

(b) How does your result show that any graph $G = (V, E)$ has a spanning bipartite subgraph $G' = (V, E')$ with $|E'| \geq |E|/2$? Explain, including how you used the probabilistic method.

(c) Design a deterministic, polynomial time algorithm for this problem, finding a spanning bipartite subgraph $G' = (V, E')$ of $G = (V, E)$, where $|E'| \geq |E|/2$.