Institut for Matematik og Datalogi
Syddansk Universitet

October 4, 2018
JFB

DM551 – Algorithms and Probability – 2018
Lecture 9

Lecture, October 1

We finished section 13.2. In section 13.3 we began with the example o0n guessing cards, since we covered the earlier part of the section earlier in the course. We finished section 13.3, and covered up through theorem 13.15 in section 13.4 in *Algorithm Design* by Kleinberg and Tardos.

Lecture, October 8

We will finish section 13.4 and cover section 13.5 in Kleinberg and Tardos. We will also analyze the expected number of comparisons done by Randomized Quicksort, using section 7.4.2 of *Introduction to Algorithms*, 3rd edition, by Cormen, Leiserson, Rivest, and Stein (CLRS).

Lecture, October 9

We will cover section 13.9 (without proofs) and section 13.10 in Kleinberg and Tardos.

Problems to be discussed on October 23

1. Do the following problem (from an earlier exam set due to Jørgen Bang-Jensen - you can find it in Danish as problem 7 at http://imada.sdu.dk/∼jbj/DM551/jan15.pdf).

   This problem concerns a randomized algorithm for coloring graphs. Let $G = (V, E)$ be a graph and let $f : V \to \{1, 2, 3, 4\}$ be a coloring of G's vertices with 4 colors (the colors are numbers between 1 and 4, and each edge is assigned one such color). Such a coloring f is called a 4-coloring. We say that an edge $uv \in E$ is *good* with respect to f if $f(u) \neq f(v)$.

   (a) Suppose that $f : V \to \{1, 2, 3, 4\}$ is a random coloring of V with 4 colors (each vertex gets color i with probability 1/4 for $i \in \{1, 2, 3, 4\}$, and each vertex is colored independently of all others). Let the random variable X be the number of edges in E that are good with respect to the coloring f. Show that $E[X] = \frac{3|E|}{4}$.

(b) Use the result above and the probabilistic method to show that every graph has a 4-coloring f such that at least $\frac{3|E|}{4}$ of its edges are good with respect to f.

(c) Define a randomized algorithm that, for a given given graph $G = (V, E)$, finds a 4-coloring f^* of V such that at least $\frac{3|E|}{4}$ of the edges of G are good with respect to f^*.

(d) What is the expected running time of your algorithm?

2. Would you use the randomized algorithm Select in Kleinberg and Tardos if you were finding the largest or smallest element in an array?

3. In CLRS, using the algorithms as presented in CLRS, (as review of Quicksort) do problems 7.1-2, 7.1-4, 7.2-3, 7.2-5.

4. In CLRS, using the algorithms as presented in CLRS, do problems 7.3-1, 7.3-2, 7.4-2, 7.4-3, 7.4-6.