Solving Linear Recurrence Relations

Definition. *Linear homogeneous recurrence relation of degree k with constant coefficients* —

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_k a_{n-k}$$

$c_i \in \mathbb{R}$ for $1 \leq i \leq k$, $c_k \neq 0$.

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Examples: Fibonacci numbers: Degree 2.

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Number of strings with n binary digits: Degree 1.

$$a_n = 2a_{n-1}$$

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Examples which are not: (linear homogeneous recurrence relations with constant coefficients)

$H_n - 2H_{n-2} + 1$ — not homogeneous

$a_n = a_{n-1} + a_{n-2}^2$ — not linear

$a_n = na_{n-1} + a_{n-2}$ — nonconstant coefficient

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Definition The *characteristic equation* of

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_k a_{n-k}$$

is

$$r^k - c_1 r^{k-1} - c_2 r^{k-2} - \cdots - c_{k-1} r - c_k = 0.$$

The solutions of this are the *characteristic roots*.