Institut for Matematik og Datalogi
Syddansk Universitet

October 19, 2020
JFB

DM551/MM851 – Algorithms and Probability – 2020
Lecture 10

Lecture, October 19, in U23

We covered section 13.2 in chapter 13 in KT, and covered memoryless guessing from the guessing cards example in section 13.3. This was the first material in section 13.3 that we have not already covered from Rosen's textbook.

Lecture, October 22, in U110

We will finish section 13.3, cover 13.4, and begin on section 13.5 in KT.

Lecture, October 27, in U55

We will finish section 13.5 in KT. Then, we will analyze the expected number of comparisons done by Randomized Quicksort, using section 7.4.2 of *Introduction to Algorithms*, 3rd edition, by Cormen, Leiserson, Rivest, and Stein (CLRS).

Problems to be discussed on November 4

1. Finish any problems not finished on October 28.
2. Do the following problem (from an earlier exam set due to Jørgen Bang-Jensen - you can find it in Danish as problem 7 at http://imada.sdu.dk/~jbj/D M551/jan15.pdf).

   This problem concerns a randomized algorithm for coloring graphs. Let $G = (V, E)$ be a graph and let $f : V \to \{1, 2, 3, 4\}$ be a coloring of G's vertices with 4 colors (the colors are numbers between 1 and 4, and each vertex is assigned one such color). Such a coloring f is called a 4-coloring. We say that an edge $uv \in E$ is *good* with respect to f if $f(u) \neq f(v)$.

   (a) Suppose that $f : V \to \{1, 2, 3, 4\}$ is a random coloring of V with 4 colors (each vertex gets color i with probability $1/4$ for $i \in \{1, 2, 3, 4\}$, and each vertex is colored independently of all others). Let the random variable X be the number of edges in E that are good with respect to the coloring f. Show that $E[X] = \frac{3|E|}{4}$.

(b) Use the result above and the probabilistic method to show that every graph has a 4-coloring f such that at least $\frac{3|E|}{4}$ of its edges are good with respect to f.

(c) Define a randomized algorithm that, for a given given graph $G = (V, E)$, finds a 4-coloring f^* of V such that at least $\frac{3|E|}{4}$ of the edges of G are good with respect to f^*.

(d) What is the expected running time of your algorithm?

3. Would you use the randomized algorithm Select in Kleinberg and Tardos if you were finding the largest or smallest element in an array?

4. In CLRS, using the algorithms as presented in CLRS, (as review of Quicksort – skip them if you remember Quicksort well enough) do problems 7.1-2, 7.1-4, 7.2-3, 7.2-5.

5. In CLRS, using the algorithms as presented in CLRS, do problems 7.3-1, 7.3-2, 7.4-2, 7.4-3, 7.4-6.