Institut for Matematik og Datalogi
Syddansk Universitet

September 23, 2020
JFB

DM551/MM851 – Algorithms and Probability – 2020
Lecture 5

Lecture, September 17

We covered up through subsection 7.2.6 of section 7.2.

Lecture, September 23

We will finish section 7.2 and cover 7.3, possibly beginning on 7.4.

Lecture, September 28

We will finish section 7.4 (Example 9 will be done using the InsertionSort algorithm on the next lecture note, along with the analysis there. This is the preferred InsertionSort.) We will finish section 7.4, with an example using Chebyshev's Inequality.

Problems to be discussed on September 30

1. Finish any problems not finished on September 25.
2. Supplementary Exercises, Chapter 7 (pages 522–523): 14, 16, 24.
3. Section 7.4: 8, 12, 24.
4. Do problem 2 in this exam set written by Jørgen Bang-Jensen in Danish:

   http://imada.sdu.dk/~jbj/DM551/jan09.pdf

   Here it is in English:

   A particle starts at time $t = 0$ at a point O (consider it point $x = 0$ on the x-axis). At every time unit, the particle either moves one unit forward (in the positive x direction) with probability p, or stays still with probability $q = 1 - p$. Let $P_n(r)$ be the probability that the particle is in position r (on the x-axis) at time $t = n \geq r \geq 0$.

   - Show that $P_n(r) = \binom{n}{r} p^r q^{n-r}$.

- Suppose now that the particle is in the (x, y)-coordinate system, and it starts at $O = (0, 0)$. Now suppose that at each time unit, it moves one unit in the positive x-direction with probability p, or moves one unit in the positive y-direction with probabilty $q = 1 - p$. What is the probability $Q_n(r, s)$ that the particle is at the coordinates (r, s) at time $t = n \geq r, s \geq 0$?
- Suppose that $p = 1/3$ and $q = 2/3$ in the above scenario (2-dimensional). What are the probabilities of the following events?
  - The particle passes through $(5, 2)$ (at some time).
  - The particle passes through $(5, 2)$ and $(7, 1)$.
  - The particle passes through $(5, 2)$ and $(6, 3)$.