Institut for Matematik og Datalogi
Syddansk Universitet

Assignment 2
Complexity and Computability — 2015

This is your second assignment in DM553. The assignment is due at 8:15 on Friday, April 10. You may write this either in Danish or English. Write your full name (or names if you do it together) clearly on the first page of your assignment (on the top, if it's not a cover page). Turn it in as a PDF file via Blackboard through your DM553 course. The assignment hand-in is in the menu for the course and is called "SDU Assignment". Keep the receipt it gives you proving that you turned your assignment in on time. Blackboard will not allow you to turn in an assignment late.

Cheating on this assignment is viewed as cheating on an exam. If you have questions about the assignment, come to Joan Boyar.

Please note that you must have this assignment approved in order to pass DM553. If it is not turned in on time, or if you do not get it approved, it will count as one of your two retries in the course, and you must have it approved on your only allowed retry for this assignment.

Assignment 2

Do the following problems. Write clear, complete answers, but not longer than necessary.

1. Design a Turing machine which decides $\{wcw^Rcw \mid w \in \{a,b\}^*\}$. Give a formal description, either with the complete transition function or with a state diagram. You may use more than one tape. Also explain in words how your Turing machine works.

2. Consider the following definition of acceptance by a Turing machine: A TM M accepts $w \in \Sigma^*$ if and only if M halts on input w. Prove that a language L is accepted by some Turing machine using this definition if and only if it is accepted by some Turing machine using the standard definition of acceptance (using states). Note that these Turing machines do not have q_{accept} and q_{reject} states.

3. Prove that if the languages L_1 and L_2 are decidable, then $L_1 \cap L_2$ is also decidable.

4. Prove that there exist languages L_1 and L_2, such that L_1 is decidable and L_2 is undecidable, where $L_1 \cap L_2$ is decidable.

5. Prove that there exist languages L_1 and L_2, such that L_1 is decidable and L_2 is undecidable, where $L_1 \cap L_2$ is undecidable.

6. Consider $L = \{\langle G, R\rangle \mid G$ is a context-free gammar and R is a regular expression, and $L(G) \cap L(R) = \emptyset\}$. Prove that L is decidable.

7. Consider $L = \{\langle M\rangle \mid M$ is a TM, and $\exists w \in \Sigma^*,$ such that M halts on input $w\}$. Prove that L is undecidable.

8. Consider $L = \{\langle M, w, q\rangle \mid M = (Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$ is a TM, $w \in \Sigma^*$, $q \in Q \backslash \{q_{accept}, q_{reject}\}$, M enters state q at some point in its computation when its input is $w\}$. Show that L is undecidable.