Institut for Matematik og Datalogi
Syddansk Universitet

Assignment 1
Complexity and Computability — 2016

This is your first assignment in DM553. The assignment is due at 9:00 on
Monday, March 7. You may write this either in Danish or English. Write
your full name (or names if you do it together — up to three people may
work together) clearly on the first page of your assignment (on the top, if
it's not a cover page). Turn it in as a PDF file via Blackboard through your
DM553 course. The assignment hand-in is in the menu for the course and
is called "SDU Assignment". Keep the receipt it gives you proving that you
turned your assignment in on time. Blackboard will not allow you to turn in
an assignment late.

Cheating on this assignment is viewed as cheating on an exam. Do not talk
with anyone outside of your group (or Christian Kudahl or Joan Boyar) about
the assignment. If you have questions about the assignment, come to Joan
Boyar or Christian Kudahl.

Please note that you must have this assignment approved in order to pass
DM553. If it is not turned in on time, or if you do not get it approved, it will
count as one of your two retries in the course, and you must have it approved
on your only allowed retry for this assignment.

Assignment 1

Do the following problems. Write clear, complete answers, but not longer
than necessary.

Note that some of the parts from problems 3 and 4 will be easier to do after
the discussion section on March 1, but most can be started on now.

1. Consider the following grammar:

$$
\begin{aligned}
S &\rightarrow AbX \\
A &\rightarrow aA \mid \epsilon \\
X &\rightarrow bcX \mid \epsilon
\end{aligned}
$$

(a) Define a DFA which recognizes the language generated by this language, using a state diagram.

(b) Convert this DFA to an equivalent GNFA. Then create an equivalent GNFA with one less state by removing a state from the GNFA which corresponds to a state in your DFA which had transitions to it from two other states. To do this, use the CONVERT procedure.

(c) Give a regular expression that expresses the language recognized by your original DFA (which should be the same as that recognized by your two GNFAs).

2. Prove that $\{uvw \mid u, v, w \in \Sigma^*, v \in L\}$ is a regular language if L is a regular language.

3. Let $L = \{a^{3i}(abc)^j(bc)^{2i} \mid i, j \geq 0\}$.

   (a) Give a context-free grammar, G, which generates L.

   (b) Show a derivation of $aaaabcabcbcbc$.

   (c) Prove that L is not regular.

   (d) Convert your grammar, G, to Chomsky Normal Form.

   (e) Define a PDA to recognize the language generated by this grammar, using a state diagram.

4. Prove that $\{xwyw \mid w, x, y \in \{a, b\}^*, |x| = |y| = 3\}$ is not context-free.

5. Prove that the following statement is false: If L_1 is a regular language and L_2 is not context-free, then $L_1 \cap L_2$ is not a regular language.