Institut for Matematik og Datalogi
Syddansk Universitet

Assignment 2
Complexity and Computability — 2016

This is your second assignment in DM553. **The assignment is due at 9:00 on Tuesday, April 12**. You may write this either in Danish or English. Write your full name (or names if you do it together) clearly on the first page of your assignment (on the top, if it's not a cover page). Turn it in as a PDF file via Blackboard through your DM553 course (only one per group). The assignment hand-in is in the menu for the course and is called "SDU Assignment". Keep the receipt it gives you proving that you turned your assignment in on time. Blackboard will not allow you to turn in an assignment late.

Cheating on this assignment is viewed as cheating on an exam. If you have questions about the assignment, come to Joan Boyar or Christian Kudahl.

Please note that you must have this assignment approved in order to pass DM553. If it is not turned in on time, or if you do not get it approved, it will count as one of your two retries in the course, and you must have it approved on your only allowed retry for this assignment.

Assignment 2

Do the following problems. Write clear, complete answers, but not longer than necessary.

1. Design a (possibly multitape) deterministic Turing machine which decides
$$\{w_1 c w_2 c w_1^R c w_2^R \mid w_1, w_2 \in \{a, b\}^*\}$$
over the alphabet $\{a, b, c\}$. Recall that w^R is w reversed. Give a formal description, either with the complete transition function or with a state diagram. **Also explain in words how your Turing machine works.**

2. Prove that if the languages L_1 and L_2 are decidable, then $L_1 \cap \overline{L_2}$ is also decidable.

3. Define languages L_1 and L_2, such that L_1 is decidable and L_2 is undecidable, where $L_1 \cap \overline{L_2}$ is decidable.

4. Define languages L_1 and L_2, such that L_1 is decidable and L_2 is undecidable, where $L_1 \cap \overline{L_2}$ is undecidable.

5. Consider $L = \{\langle P, R\rangle \mid P$ is a pushdown automaton, R is a regular expression, and $L(P) \cap L(R) = \emptyset\}$. Prove that L is decidable.

6. Consider $L = \{\langle M, w, q\rangle \mid M = (Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$ is a TM, $w \in \Sigma^*$, $q \in Q \setminus \{q_{accept}, q_{reject}\}$, M enters state q at some point in its computation when its input is $w\}$. Show that L is undecidable.

7. Use Rice's Theorem to show that

$$\{\langle M\rangle \mid M \text{ is a TM and } |L(M)| \text{ is odd}\}$$

is undecidable.