Institut for Matematik og Datalogi
Syddansk Universitet

Assignment 3
Complexity and Computability — 2016

This is your third assignment in DM553. **The assignment is due at 9:00 on Wednesday, April 27**. You may write this either in Danish or English. Write your full name (or names if you do it together) clearly on the first page of your assignment (on the top, if it's not a cover page). Turn it in as a PDF file via Blackboard through your DM553 course (only one per group). The assignment hand-in is in the menu for the course and is called "SDU Assignment". Keep the receipt it gives you proving that you turned your assignment in on time. Blackboard will not allow you to turn in an assignment late.

Cheating on this assignment is viewed as cheating on an exam. If you have questions about the assignment, come to Joan Boyar or Christian Kudahl.

Please note that you must have this assignment approved in order to pass DM553. If it is not turned in on time, or if you do not get it approved, it will count as one of your two retries in the course, and you must have it approved on your only allowed retry for this assignment.

Assignment 3

In the game Cardstone (this assignment idea is due to Christian Kudahl), two players have monsters battling each other. A *state* in the game is represented by two arrays, one for you and one for your opponent. Your array is called A, your opponent's array is called B.

Array A contains all your monsters (one in each space). Array B contains all your opponent's monsters (also one in each space). A monster is represented by an attack value and a health value separated by a slash. A monster with attack value 3 and health value 4 would, for example, be represented as 3/4.

On your turn, each of your monsters may attack (no more than once each). You may decide the order in which they attack. A monster attacks one

Figure 1: The top array is yours. The bottom one is your opponent's.

2/1	2/10	3/1	2/2	3/4

1/1	2/7	5/3	4/1

opponent's monster of your choice. When it does, they both have their health reduced by the attack value of the monster they fight.

As an example, let monster a be represented by $2/3$ and let monster b be represented by $1/1$. If a attacks b (or b attacks a), a will be a $2/2$ monster after the attack and b will be a $1/-1$. If a monster's health is reduced to 0 or below, it dies and is removed from the game.

The Board-clear problem we consider is the following: Given a state in the game, is it possible for you to kill all the opponent's monsters in a single turn? This can be written as a language acceptance problems as:

$$\text{Board-clear} = \{\langle A, B\rangle \mid \langle A, B\rangle \text{ is a state in Cardstone where it is possible for the monsters in } A \text{ to kill all opposing monsters in } B \text{ in a single turn}\}$$

Do the following problems concerning Cardstone: Write clear, complete answers, but not longer than necessary.

1. Consider the state in Figure 1. Is it possible for you to kill all the opponent's monsters in a single turn? Explain your answer.

2. Show the the Board-clear problem is NP-complete.
   Hint: Reduce from the Subset Sum Problem.

3. Suppose that the arrays A and B in the Board-clear problem never have more than length 20 (no more than 20 monsters each). Is the problem still NP-Complete? Prove your answer.

4. Suppose that you have a polynomial-time algorithm for solving the Board-clear problem. Describe how to use this algorithm to find an ordering of attacks (which monster in A attacks which monster in B when) of monsters in A to kill all monsters in B in a single turn (when this is possible). Argue that your algorithms would be polynomial time. Note that this shows that the optimization version of this problem (the problem of finding an attack order) is also NP-hard.