Institut for Matematik og Datalogi
Syddansk Universitet

Exam Assignment 1
Complexity and Computability — 2016

This is the first of three sets of problems (assignments) which together with the oral exam in June constitute the exam in DM553. This first set of problems may be solved in groups of up to three.

The assignment is due at 10:15 on Tuesday, March 13. You may write this either in Danish or English. Write your full name (or names if you do it together — up to three people may work together) clearly on the first page of your assignment (on the top, if it's not a cover page). Turn it in as a PDF file via Blackboard through your DM553 course. The assignment hand-in is in the menu for the course and is called "SDU Assignment". Keep the receipt it gives you proving that you turned your assignment in on time. Blackboard will not allow you to turn in an assignment late.

Cheating on this assignment is viewed as cheating on an exam. Do not talk with anyone outside of your group (or David Hammer or Joan Boyar) about the assignment, and do not show your solutions to anyone outside your group. If you have questions about the assignment, come to Joan Boyar or David Hammer.

Assignment 1

Do the following problems. Write clear, complete answers, but not longer than necessary.

1. Consider the following grammar:

$$\begin{aligned} S &\rightarrow BC \\ B &\rightarrow Bab \mid b \\ C &\rightarrow cC \mid \epsilon \end{aligned}$$

   (a) Define a DFA which recognizes the language generated by this language, using a state diagram.

(b) Convert this DFA to an equivalent GNFA. Then create an equivalent GNFA with one less state by removing a state from the GNFA which corresponds to a state in your DFA which had transitions to it from two other states. To do this, use the CONVERT procedure.

(c) Give a regular expression that expresses the language recognized by your original DFA (which should be the same as that recognized by your two GNFAs).

(d) Convert your regular expression to an NFA which accepts the same language, using the techniques from Lemma 1.55. Show the steps you go through.

(e) Convert your NFA back to a DFA using the technique from Theorem 1.39. Show the steps you go through. (You do not need to show all transitions as long as you show that you know how to do it, including the transitions for some new states composed of more than one original state.)

2. Prove that $\{uvw \mid u, v, w \in \Sigma^*, v, w \in L\}$ is a regular language if L is a regular language.

3. Let $L = \{(xy)^{3i}(xyz)^j(x)^i \mid i, j \geq 0\}$.

   (a) Give a context-free grammar, G, which generates L.

   (b) Show a derivation of $xyxyxyxyxyxyzxx$.

   (c) Is G ambiguous? Why or why not?

   (d) Prove that L is not regular.

   (e) Convert your grammar, G, to Chomsky Normal Form. Show the steps you go through.

   (f) Define a PDA to recognize the language L.

4. Prove that $\{xwyw \mid w, x, y \in \{0,1\}^*, |x| = 2, |y| = 3\}$ is not context-free.

5. Prove that the following statements are false:

   - If L_1 is a regular language and L_2 is not context-free, then $L_1 \cup L_2$ is not a regular language.

- If L_1 is a regular language and L_2 is not context-free, then $L_1 \cup L_2$ is a regular language.