Institut for Matematik og Datalogi
Syddansk Universitet

Exam Assignment 1
Complexity and Computability — 2020

This is the first of three sets of problems (assignments) which together with the oral exam in June constitute the exam in DM553. This first set of problems may be solved in groups of up to three.

The assignment is due at 23:59 on Sunday, March 15. You may write this either in Danish or English. Write your full name (or names if you do it together — up to three people may work together) clearly on the first page of your assignment (on the top, if it is not a cover page). Turn it in as a PDF file via Blackboard through your DM553 course. The assignment hand-in is in the menu for the course and is called "SDU Assignment". Keep the receipt it gives you proving that you turned your assignment in on time. Blackboard will not allow you to turn in an assignment late.

Cheating on this assignment is viewed as cheating on an exam. Do not talk with anyone outside of your group (except Joan Boyar or David Hammer) about the assignment, and do not show your solutions to anyone outside your group. If you have questions about the assignment, come to Joan Boyar or David Hammer.

Assignment 1

Do the following problems. Write clear, complete answers, but not longer than necessary.

Note that some of the problems relate to material that will not be covered in discussion seciton until after this assignment is posted, but it will be covered long before the assignment is due.

1. Let $L = \{(ab)^{3i}(ba)^j(c)^i \mid i, j \geq 0\}$ over the alphabet $\{a, b, c\}$.
   (a) Give a context-free grammar, G, which generates L.
   (b) Show a derivation of *abababababbacc*.

(c) Prove that L is not regular.

(d) Convert your grammar, G, to Chomsky Normal Form. Show the steps you go through.

(e) Define a PDA to recognize the language L. Explain how your PDA works in words, too.

2. Use the technique in Theorem 2.20 to produce a PDA which accepts the language generated by the following grammar: $G = (V, \Sigma, R, S)$, where $V = \{S, T\}$, $\Sigma = \{0, 1\}$, and R is the set of rules:

$$\begin{aligned} S &\rightarrow 00S0 \mid 1S11 \mid T \mid \epsilon \\ T &\rightarrow 0S0 \mid \epsilon \end{aligned}$$

3. Prove that $\{xwyw \mid w, x, y \in \{0,1\}^*, |x| = 3, |y| = 2\}$ is not context-free.

4. Prove that the following statements are false:

(a) If L_1 is a regular language and L_2 is not context-free, then $L_1 \cap L_2$ is not a regular language.

(b) If L_1 is a regular language and L_2 is not context-free, then $L_1 \cap L_2$ is a regular language.

5. Design a (possibly multitape) deterministic Turing machine which decides

$$\{w_1^R c w_2^R d w_1 e w_2 \mid w_1, w_2 \in \{a, b\}^*\}$$

over the alphabet $\{a, b, c, d, e\}$. Recall that w^R is w reversed. Give a formal description, either with the complete transition function or with a state diagram. (For the state diagram for a 2-tape Turing machine, you can write the symbols being read and directions being moved as pairs and otherwise do it as with a single tape Turning machine.) **Also explain in words how your Turing machine works.**