Institut for Matematik og Datalogi
Syddansk Universitet

Exam Assignment 2
Complexity and Computability — 2020

This is the second of three sets of problems (assignments) which together with the oral exam in June constitute the exam in DM553/MM850. This second set of problems must be solved individually, not in groups.

The assignment is due at 23:59 on Wednesday, April 15. You may write this either in Danish or English. Write your full name clearly on the first page of your assignment (on the top, if it's not a cover page). Turn it in as a PDF file via Blackboard through your DM553 course. The assignment hand-in is in the menu for the course and is called "SDU Assignment". Keep the receipt it gives you proving that you turned your assignment in on time. Blackboard will not allow you to turn in an assignment late.

Cheating on this assignment is viewed as cheating on an exam. Do not talk with anyone except Joan Boyar or David Hammer about the assignment, and do not show your solutions to anyone. If you have questions about the assignment, come to Joan Boyar or David Hammer.

Assignment 2

Do the following problems. Write clear, complete answers, but not longer than necessary.

1. For proving the undecidability of the Post Correspondence Problem, the notation $\star u$ is defined on page 233 of Sipser's textbook. Consider the alphabet $\{0, 1, 2\}$. What string does $\star 00122100$ represent over the alphabet $\{0, 1, 2, *\}$?

2. Consider the following definition of acceptance by a Turing machine: A TM M accepts $w \in \Sigma^*$ if and only if M halts on input w, and it halts on input w if and only if during its execution with w as input, it at some point is enters any state q, with the tape head reading any symbol

$a \in \Gamma$, where $\delta(q, a)$ is undefined (so at some point no transistion is defined). Thus, rather than having q_{accept} and q_{reject} states, it has some transitions which are undefined. Prove that a language L is accepted by some Turing machine using this definition if and only if it is accepted by some Turing machine using the standard definition of acceptance from Sipser's textbook (using states).

3. Suppose that L_1 and L_2 are context-free languages. Show that

$$L = \{\langle G, w\rangle \mid G \text{ is a CFG that generates string } w \in \overline{L_1 \cup L_2}\}$$

   is decidable. (The bar over $L_1 \cup L_2$ indicates the complement of $L_1 \cup L_2$.)

4. Show that A is decidable if and only if $A \leq_m \{a, b, aa, ab, ba, bb\}$ over the alphabet $\{a, b\}$.

5. Use a technique similar to that in Theorem 5.3 in Sipser to show that

$$\textit{Context-Free}_{TM} = \{\langle M\rangle \mid M \text{ is a TM and } L(M) \text{ is a context-free language}\}$$

   is undecidable.

6. Use Rice's Theorem to show that

$$\{\langle M\rangle \mid M \ \text{ is a TM and } |L(M)| \geq 10\}$$

   is undecidable.