Institut for Matematik og Datalogi
Syddansk Universitet

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JFB

DM553/MM850 – Complexity and Computability 2020 – Lecture 17

Lecture, April 23

I covered theFPTAS for Subset-Sum from section 35.5 of CLRS, and the linear programming relaxation with rounding for Vertex Cover from section 35.4.

The videos are available in Course Recordings in Blackboard, and the slides are in Course Materials.

Lecture, April 28

First I covered some randomized approximation algorithms for MAX-SAT, including the one in section 35.4 of the textbook.

Then, I began on parameterized aalgorithms from the parts of the relevant textbook that can be found in Course Materials in Blackboard.

I will be on Discord for a chat session from 10:15 to 12:00. This time the channel will be “intro-fpt”.

Lecture, May 4

I will finish fixed parameter tractability from the portions of the textbook in Course Materials in Blackboard. Then there will be an introduction to heuristics.

Problems to be discussed on May 12

1. Do any problems from earlier weeks which have not been finished.
2. 35.4-1 in CLRS.
3. 35.4-2 in CLRS (just show that the algorithm for MAX-3-SAT (MAX-3-CNF) is 2-approximate in general).
4. For Vertex Cover, consider the parameter Δ, the maximum degree of any vertex in the graph. How good an approximation algorithm can you get, as a function of Δ?
5. Suppose you have an exact algorithm for Vertex Cover. Can you use it for Independent Set? For Clique?

6. Suppose you have an approximation algorithm for Vertex Cover. Can you use it for Independent Set? For Clique?

7. Consider the problem MAX-SAT, which has a Boolean formula in CNF form and an integer k as input, asking if there is an assignment satisfying at least k clauses.

   - Is this NP-Complete?
   - Suppose $k = m/2$, where m is the number of the clauses? Can you always answer "yes" to such an instance?
   - Consider the variable-clause incident graph of a formula ϕ. This is a bipartite graph $G = (X, Y, E)$, where the vertices in X are the variables in ϕ, the vertices in Y are the clauses in ϕ, and there is an edge between a vertex in X and a vertex in Y if the variable appears in the clause (either complemented or not). Suppose there is a matching of size t in G. Use this to give a lower bound on the number of clauses that can be satisfied. (See problem 35-4 in CLRS for definitions of matchings.)
   - Hall's Theorem says that if a bipartite graph does not have a maximum matching of size the number of vertices in the smaller of X and Y, say X, there is a subset $C \subseteq X$ such that $N(C)$, the neighbors of the vertices in C, is such that $|N(C)| < |C|$. Suppose C is such a subset. Show that all clauses in $N(C)$ are satisfied in any truth assignment satisfying the maximum number of clauses in ϕ.
   - Use the previous result to give a reduction rule for MAX-SAT.
   - Conclude that you can obtain a kernel with at most k variables and $2k$ clauses.