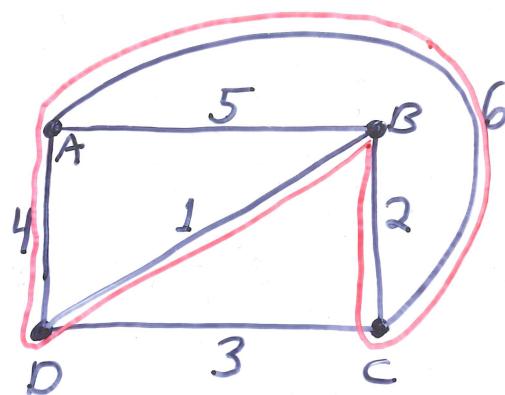


Traveling Salesman Problem



$$k = 14$$

A	B	C	D
A	5	6	4
B	5	2	1
C	6	2	3
D	4	1	3

Minimum cost tour: $ACBDA$

$$\text{cost} : 13 \leq k$$

TSP is NP-Complete

Reduction from

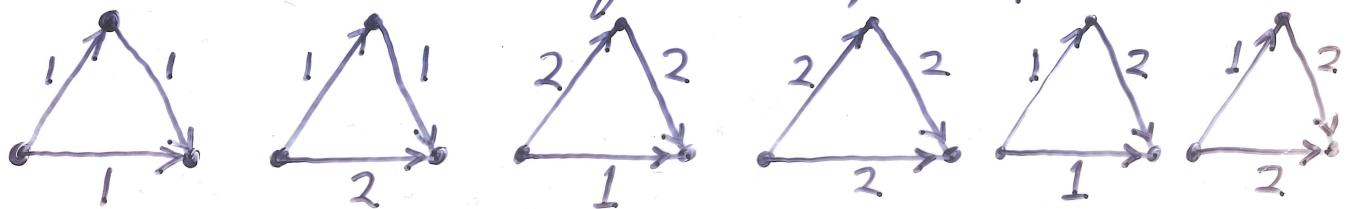
Hamiltonian Circuit.

HC Instance: An undirected graph given by an adjacency matrix, $n \times n$.
 $(1 \Rightarrow \text{edge exists}; 0 \Rightarrow \text{no edge})$

Reduction: Change all 0's to 2's.
Let $k = n$.

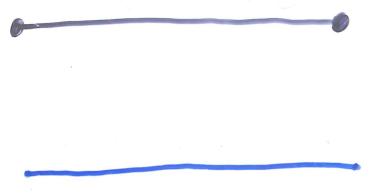
Claim: There is a Hamiltonian circuit in the original graph iff there is a tour of cost $\leq k$ in the new matrix.

Note: The triangle inequality holds.

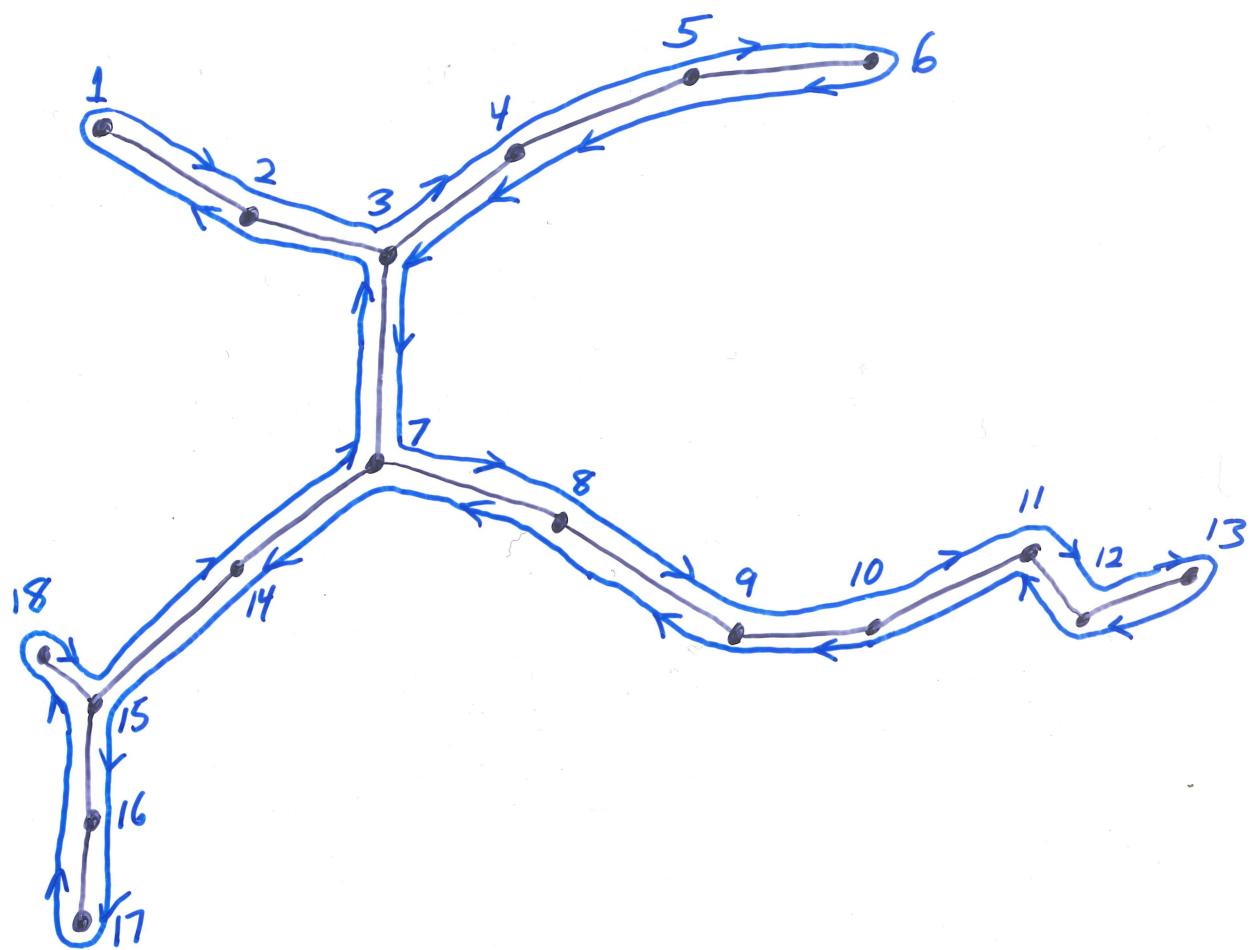


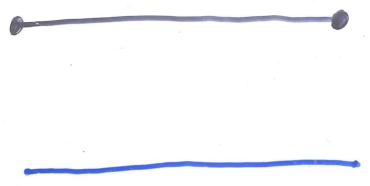
Approximation Algorithm for Δ TSP

- a deterministic polynomial-time algorithm which finds a tour with cost $\leq 2 \times (\text{optimal cost})$

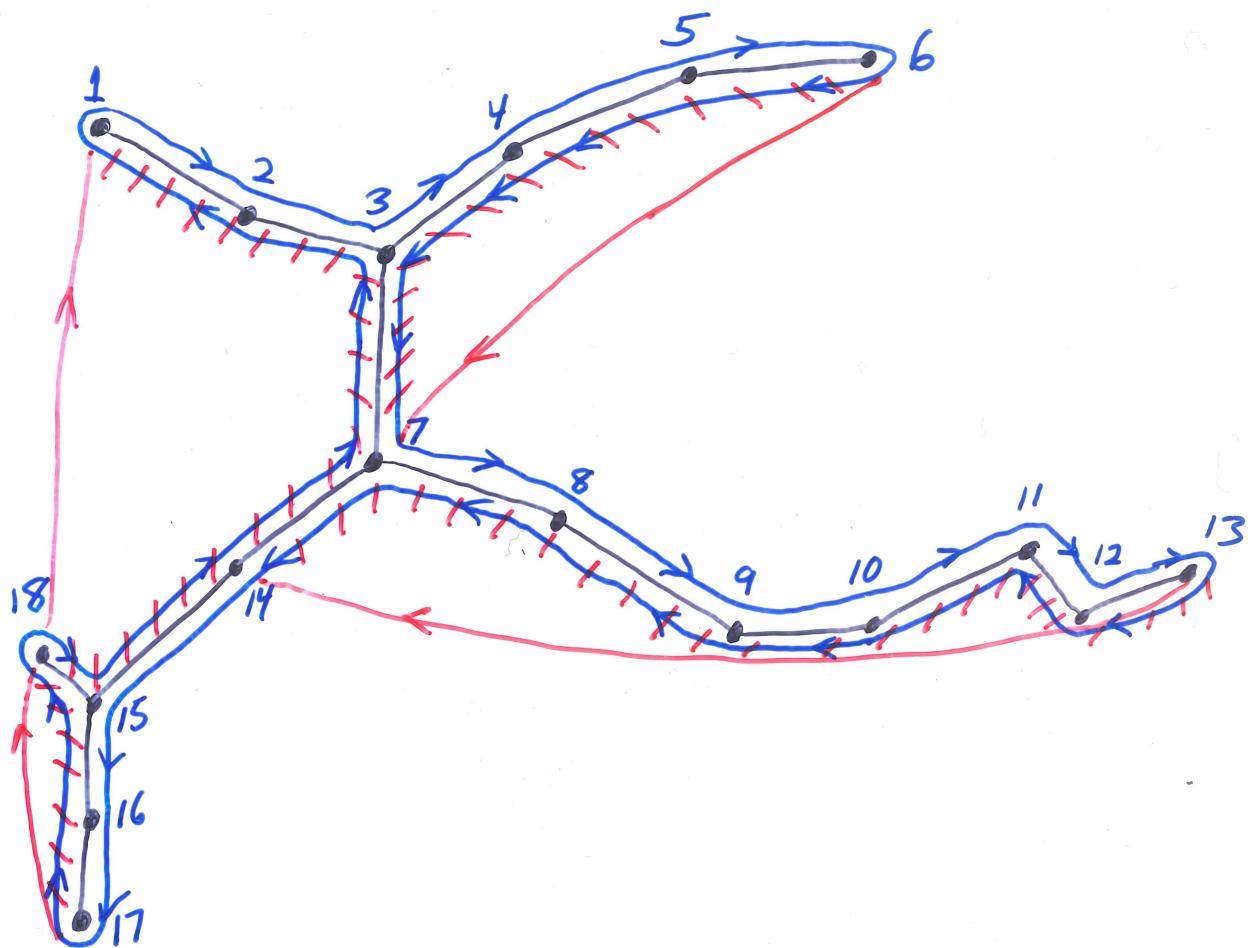


Minimum spanning tree
Depth-first search





Minimum spanning tree
Depth-first search



Step 1: Find a minimum spanning tree, T .

Step 2: Do a depth-first search of T , numbering the vertices.

Step 3: Create the tour \hat{T} defined by this numbering.

Claim: The cost of \hat{T} is $\leq 2 \times (\text{optimal cost})$.

Fact 1 \hat{T} has cost ≤ 2 times the cost of T , the minimum spanning tree.

Pf Use the Δ inequality. ■

Fact 2: T has cost \leq the cost of the optimal tour.

Pf Suppose C is an optimal tour. Remove one edge from C to get \hat{C} , a spanning tree. Since T is a minimum spanning tree, it has cost \leq that of \hat{C} , which is \leq that of C . ■

Thm There is a deterministic polynomial-time approximation algorithm for the TSP, which finds a tour with cost no more than twice the optimal, on instances such that:

- 1.) the matrix is symmetric
- 2.) the Δ -inequality holds
- 3.) the costs are non-negative.

Thm (Christofides) The factor 2 can be improved to 1.5.

- Uses matching and an Eulerian circuit.