

Def An instance of an optimization problem² is a pair (F, c) , where F is any set, the domain of feasible points, and c is the cost function, a mapping $c: F \rightarrow \mathbb{R}$. The problem is to find an $f \in F$ s.t. $c(f) \leq c(y) \forall y \in F$. f is called globally optimal (optimal).

Def An optimization problem is a set I of instances of an optimization problem. If the sets F are all finite, it is a combinatorial optimization problem.

Def Given an optimization problem w/ instances (F, c) , a neighborhood is a mapping $N: F \rightarrow 2^F$ defined for each instance.

Ex 1. $F = \mathbb{R}^n$, use Euclidean distance for nbrhd.

Ex 2 TSP - 2-change

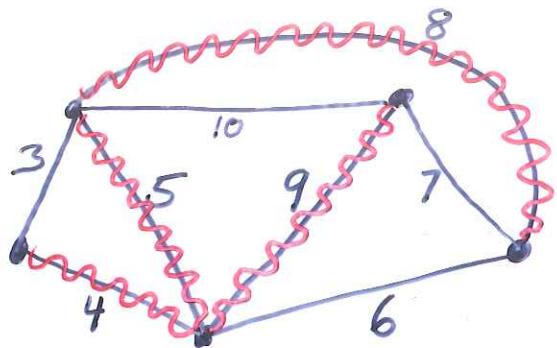
$N_2(f) = \{g \mid g \in F \text{ and } g \text{ can be obtained from } f \text{ by removing 2 edges from } f \text{ and replacing them with 2 edges}\}$

$$N_k(f) = ?$$

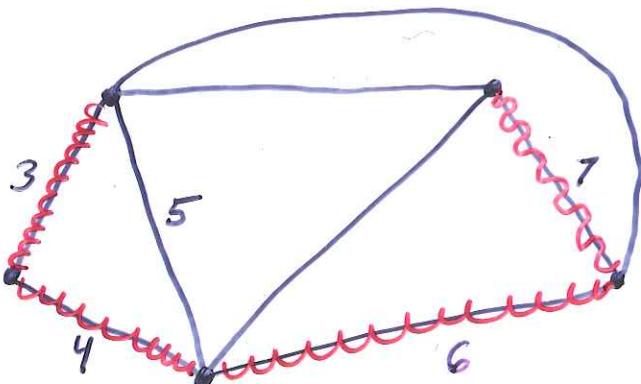
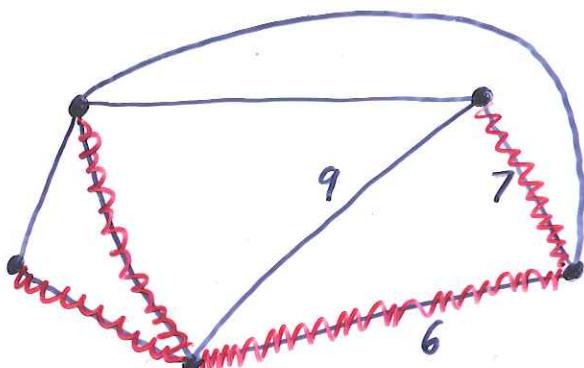
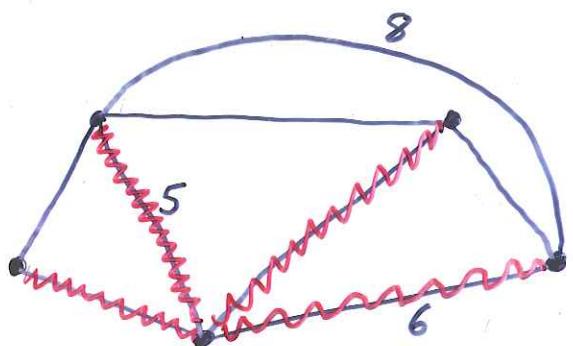


Ex 3 MST

$N(f) = \{g \mid g \in F \text{ and } g \text{ can be obtained from } f \text{ by adding an edge to } f, \text{ and deleting any edge from the cycle produced}\}$

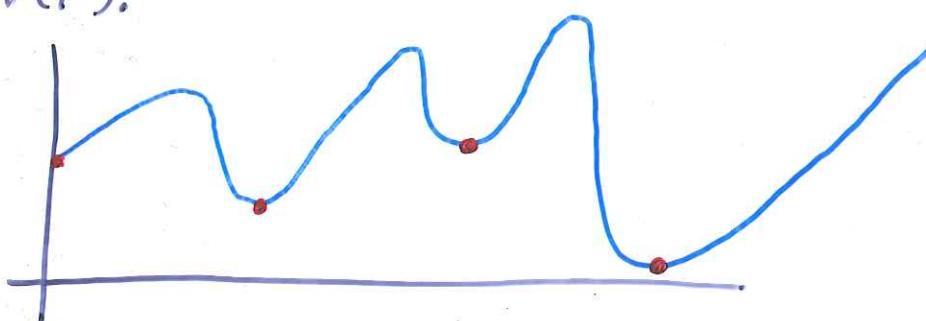


MST



Weight = 20

Def Given an instance (F, c) of an optimization problem on a neighborhood N , a feasible solution $f \in F$ is called locally optimal w.r.t. N (locally optimal) if $c(f) \leq c(g)$ $\forall g \in N(f)$.



Def If, whenever $f \in F$ is locally optimal w.r.t. N , it is also globally optimal, we say the nbrhd N is exact.

Nonlinear programming problem:

$$\begin{aligned} & \text{minimize } f(x) \\ & \text{subject to } g_i(x) \geq 0 \quad i = 1, \dots, m \\ & \qquad \qquad h_j(x) = 0 \quad j = 1, \dots, p_n \end{aligned}$$

where f, g_i, h_j are general fnctns of $x \in \mathbb{R}^n$.

Techniques - iterative

Convergence - studied using real analysis

When f is convex, g_i concave, and h_j linear, convex programming problem.

local optimality \rightarrow global optimality
sufficient conditions for optimality
- Kuhn-Tucker conditions