

Files

Sequential Access

Random Access

Files

Sequential Access

Merge

MergeSort

Analysis

2 standard methods for accessing data:

- sequential access
- random access: access via index or ID (key) for data element

Sequential access API

Files

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(API = Application Programming Interface: collection of methods).

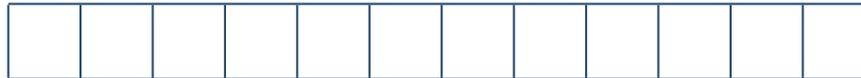
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(API = Application Programming Interface: collection of methods).

Reading:

Operations: `readNext()`, `isEndOfFile()`, `open()`, `close()`



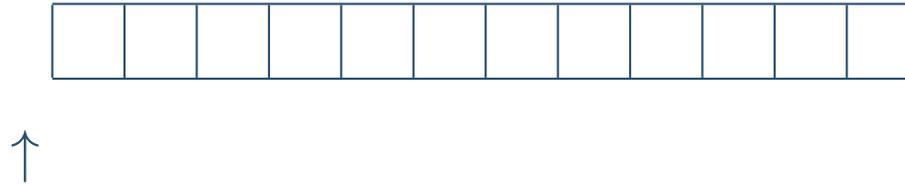
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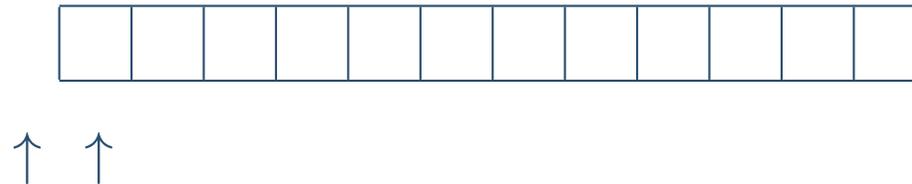
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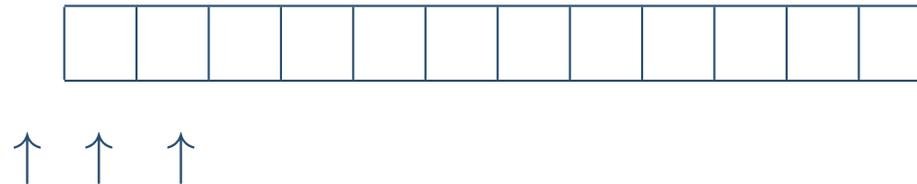
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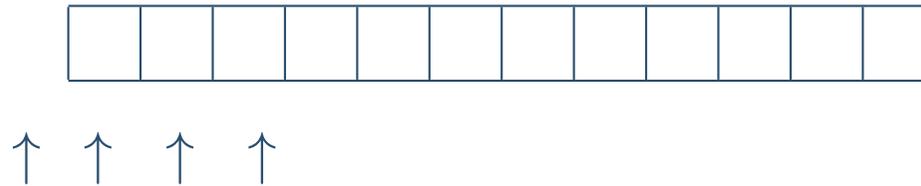
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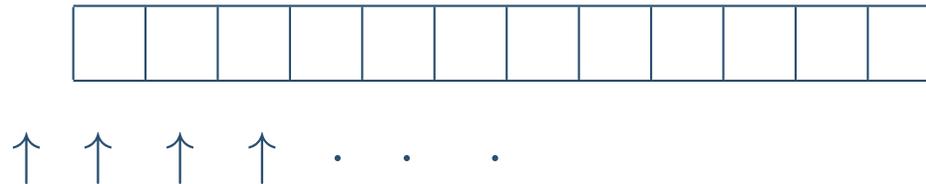
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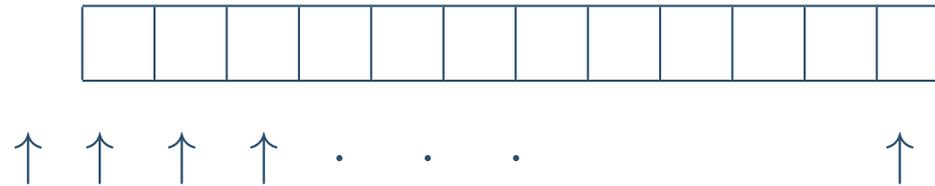
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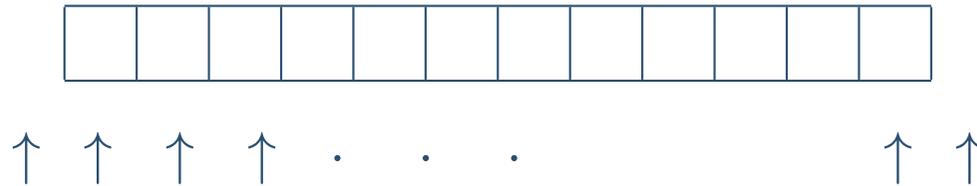
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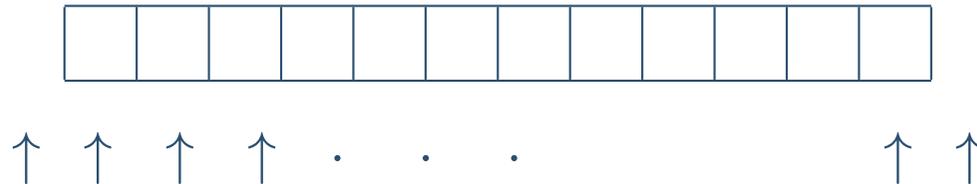
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Writing:

Operations: `writeNext()`, `open()`, `close()`



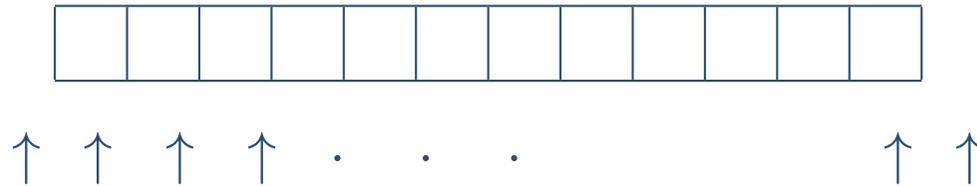
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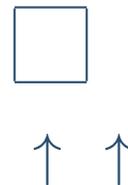
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Operations: `writeNext()`, `open()`, `close()`



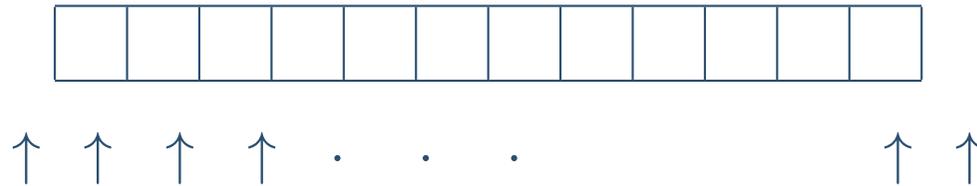
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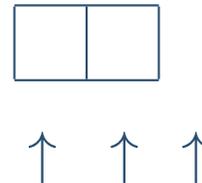
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Writing:

Operations: `writeNext()`, `open()`, `close()`



Random access API

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random access: access via ID (key) for data element

Operations:

```
findElm(ID)
insertElm(ID,elementData)
deleteElm(ID)
open()
close()
```

Examples:

- dictionaries in Python
- arrays in Java — with ID = index in array

Questions

Files
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1. What can be done using only Sequential access?
2. How can one implement Random access?

Importance

Most data sources can be accessed by Sequential access.
Some can only be accessed sequentially.

- hard disk
- CD, DVD
- tape
- streaming (over the Internet)
- data generated on-the-fly, by another program
- data in an array

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What can be done using only Sequential access?

What can be done using only Sequential access?

- Sequential search?
- Insertion Sort?
- Find maximum entry?
- Selection Sort?
- Find sum and average?

Find sum and average

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```
procedure SumAverage( $A$ )  
  open( $A$ )  
   $(s, n) \leftarrow$  SumAve( $A, 0, 0$ )  
  close( $A$ )  
  return( $s, s/n$ )
```

```
procedure SumAve( $A, s, n$ )  
  if (isEndOfFile( $A$ )) then  
    return( $s, n$ )  
  else  
    SumAve( $A, s + \text{readNext}(\mathbf{A}), n + 1$ )
```

Find sum and average

Invariants: s is sum of first n entries of A .

Next entry of A is the $n + 1$ st.

Therefore, SumAverage computes the correct result.

Fundamental operation: readNext —
done $\text{length}(A)$ times, once in each recursive call, except n th.

Find sum and average

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```
procedure SumAverage( $A$ )
  open( $A$ )
   $(s, n) \leftarrow$  SumAve( $A, 0, 0$ )
  close( $A$ )
  if  $(n > 0)$  return  $(s, s/n)$ 
  else report empty file
```

```
procedure SumAve( $A, s, n$ )
  if (isEndOfFile( $A$ )) then
    return  $(s, n)$ 
  else
    SumAve( $A, s + \text{readNext}(A), n + 1$ )
```

What can be done using only Sequential access?

- Sequential search?
- Insertion Sort?
- Find maximum entry?
- Selection Sort?
- Find sum and average?
- Merging 2 sorted lists into 1?
- Mergesort?

Merging 2 lists

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Input: 2 lists, A and B are both sorted.

Output: 1 sorted list, C , containing the entries of A and B .

Merging 2 lists

Input: 2 lists, A and B are both sorted.

Output: 1 sorted list, C , containing the entries of A and B .

Merge Step:

- Compare current records of A and B .
- Put smallest in C .
- Advance to next record in A or B , whichever had that smallest entry.

Merging 2 lists

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```
procedure MergeFiles(A, B, C):
    open(A); open(B); open(C); fA,fB,fC ← false;
    if (isEndOfFile(A) and isEndOfFile(B)) then Stop with C empty
    if (not isEndOfFile(A)) then currentA ← readNext(A); fA ← true;
    if (not isEndOfFile(B)) then currentB ← readNext(B); fB ← true;
    while (fA and fB) do
        if (currentA ≤ currentB) then
            writeNext(currentA,C)
            if (not isEndOfList(A)) then currentA ← readNext(A)
            else fA ← false
        else
            writeNext(currentB,C)
            if (not isEndOfList(B)) then currentB ← readNext(B)
            else fB ← false
    Starting with the current record in the input file which is not at EOF,
        copy the remaining records to C
    close(A); close(B); close(C)
```

Merging 2 arrays

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procedure MergeArrays(A, B, C):

$i, j, k \leftarrow 1$

if ($\text{length}(A) = 0$ and $\text{length}(B) = 0$) **then** Stop with C empty

if ($\text{length}(A) > 0$) **then** $\text{currentA} \leftarrow A[1]$

if ($\text{length}(B) > 0$) **then** $\text{currentB} \leftarrow B[1]$

while ($\text{length}(A) \geq i$ and $\text{length}(B) \geq j$) **do**

 { **Merge Step**() }

if ($\text{currentA} \leq \text{currentB}$) **then**

$c[k] \leftarrow \text{currentA}; k \leftarrow k + 1; i \leftarrow i + 1;$

if ($\text{length}(A) \geq i$) **then** $\text{currentA} \leftarrow A[i]$

else

$c[k] \leftarrow \text{currentB}; k \leftarrow k + 1; j \leftarrow j + 1;$

if ($\text{length}(B) \geq j$) **then** $\text{currentB} \leftarrow B[j]$

Starting with the current record in the array which is not finished,
copy the remaining records to C

Merge Sort

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procedure MergeSort(A, f, l):
 { Input: Array A with first index f and last index l }
 { Output: Sorted array, A , with same entries as input A }

if ($f < l$) **then**
 $m \leftarrow (f + l) \text{ div } 2$
 MergeSort(A, f, m)
 MergeSort($A, m + 1, l$)
 MergeArrays($A[f..m], A[m + 1..l], C$)
 Copy C to A

MergeSort($A, 1, \text{length}(A)$);

Analysis of Merge Sort

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Let $T(n)$ be the maximum number of comparisons MergeSort uses if $\text{length}(A) = n$.

Let $M(m_A, m_B)$ be the maximum number of comparisons MergeArrays uses if $\text{length}(A) = m_A$ and $\text{length}(B) = m_B$.

$$T(n) \leq T(\lceil \frac{n}{2} \rceil) + T(\lfloor \frac{n}{2} \rfloor) + M(\lceil \frac{n}{2} \rceil, \lfloor \frac{n}{2} \rfloor)$$

Need to calculate $M(m_A, m_B)$.

Analysis of Merge Sort

As an aside, what is the minimum number of comparisons done by MergeArrays, given m_A and m_B ?

A. $\min(m_A, m_B) - 1$

B. $\min(m_A, m_B)$

C. $\max(m_A, m_B) - 1$

D. $\max(m_A, m_B)$

E. $m_A + m_B - 1$

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Analysis of Merge Sort

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What is $M(m_A, m_B)$?

A. $m_A + m_B - 1$

B. $m_A + m_B$

C. $m_A + m_B + 1$

D. $m_A \cdot m_B$

E. $m_A \cdot m_B + 1$

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Analysis of Merge Sort

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$$\begin{aligned} T(n) &\leq T\left(\left\lceil\frac{n}{2}\right\rceil\right) + T\left(\left\lfloor\frac{n}{2}\right\rfloor\right) + M\left(\left\lceil\frac{n}{2}\right\rceil, \left\lfloor\frac{n}{2}\right\rfloor\right) \\ &\leq T\left(\left\lceil\frac{n}{2}\right\rceil\right) + T\left(\left\lfloor\frac{n}{2}\right\rfloor\right) + \left(\left\lceil\frac{n}{2}\right\rceil + \left\lfloor\frac{n}{2}\right\rfloor - 1\right) \\ &\leq T\left(\left\lceil\frac{n}{2}\right\rceil\right) + T\left(\left\lfloor\frac{n}{2}\right\rfloor\right) + n - 1 \end{aligned}$$

$$T(n) \in \Theta(n \log n).$$