Random access API

random access: access via ID (key) for data element

Operations:
  - findElm(ID)
  - insertElm(ID, elementData)
  - deleteElm(ID)
  - open()
  - close()

Examples:
  - dictionaries in Python
  - arrays in Java — with ID = index in array
Random access API

How do you implement random access?

One solution: hashing.
Hashing

Idea:

- store values in an array $A$
- ID determines index where stored

Hash function: $h$

$h(ID) = \text{index in } A$
Hashing

Hash function: \( h \)

\[ h(ID) = \text{index in } A \]

Example: Assume ID is an integer, \(|A| = n\).

\[ h(ID) = ID \mod n \]

Note: \( h(ID) \in \{0, 1, 2, \ldots, n - 1\} \), so legal index.
Hashing

Hash function: \( h \)

Example: Assume ID is an integer, \(|A| = k\).

\[
    h(ID) = ID \pmod{k}
\]

Note: \( h(ID) \in \{0, 1, 2, \ldots, k - 1\} \), so legal index.

Let \( k = 41 \).

\[
\begin{align*}
    h(46) &= 5 \quad \text{since } 1 \cdot 41 + 5 = 46 \\
    h(12) &= 12 \quad \text{since } 0 \cdot 41 + 12 = 12 \\
    h(100) &= 18 \quad \text{since } 2 \cdot 41 + 18 = 100 \\
    h(479869) &= 5 \quad \text{since } 11704 \cdot 41 + 5 = 479869
\end{align*}
\]
Hashing

Why not let $h(x) = x$?

Example: IDs (keys) are CPR-numbers.

CPR-number: 180796-2345

$\in \{0, 1, 2, \ldots, 10^{10} - 1\}$

$10^{10}$ bytes $> 9$ GB (just to store one byte per key).

Huge waste of space!

Suppose you only need to store 6 million records.

You need to allocate space for $10^{10}$ records.

You are allocating more than 1000 records for every one used.
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If the keys are 64-bit integers...
Collisions

Let $k = 41$.

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1st solution: Chaining

For each cell in array, have a linked list for elements stored there.
1st solution: Chaining

Assume computing $h(x)$ takes constant time.

Worst case: How long does it take to find a record from a key if there are no collisions?

How long does it take if there are at most $s$ collisions for any cell?

A. $\Theta(1), \Theta(1)$.
B. $\Theta(1), \Theta(k)$.
C. $\Theta(1), \Theta(s)$.
D. $\Theta(k), \Theta(s)$.
E. $\Theta(k), \Theta(k \cdot s)$.

Vote at m.socrative.com. Room number 415439.
1st solution: Chaining

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C. $\Theta(1)$, $\Theta(s)$.

So we want short lists, few collisions.
Can collisions be avoided?

Suppose the hash function $h$ is fixed.

Suppose the total number of items in the domain of $h$ is $d$ and the array has size $k$. 
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Then $\left\lceil \frac{d}{k} \right\rceil$ elements would hash to some cell.

It depends on the relation between the hash function and the data set.
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Then \( \lceil \frac{d}{k} \rceil \) elements would hash to some cell.
It depends on the relation between the hash function and the data set.

In the worst case all \( n \) elements being hashed go to the same cell.
Time: \( \Theta(n) \).
Can collisions be avoided?

If \( n \) (number of elements hashed) > \( k \) (size of array), there is at least one collision (Pigeon Hole Principle).

The best hash functions “appear” to hash numbers to random cells.
The birthday paradox

**Situation:** There are $n$ random people in a room.

**Question:** Are there two that have the same birthday? (Ignore year.)
The birthday paradox

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Question: Are there two that have the same birthday? (Ignore year.)

<table>
<thead>
<tr>
<th>$n$</th>
<th>Probability for 2 with same birthday</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>$\frac{1}{365}$</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>366</td>
<td>1</td>
</tr>
</tbody>
</table>

Question: For which $n$ is the probability $\geq 1/2$?
The birthday paradox

Let \( s_n \) = probability none of \( n \) have same birthday.

\[
s_n = s_{n-1} \cdot \frac{365 - (n - 1)}{365}
\]

Note: \( s_1 = 1 \).

<table>
<thead>
<tr>
<th>( n )</th>
<th>( s_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1 \cdot \frac{364}{365}</td>
</tr>
<tr>
<td>3</td>
<td>1 \cdot \frac{364}{365} \cdot \frac{363}{365}</td>
</tr>
<tr>
<td>4</td>
<td>1 \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365}</td>
</tr>
<tr>
<td>\vdots</td>
<td>\vdots</td>
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Computing these $s_n$ gives:

\[ s_{22} = 0.5243... \]
\[ s_{23} = 0.4972... \]

So when is the probability $\geq 1/2$ that 2 have the same birthday?
The birthday paradox

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\[ s_{23} = 0.4972... \]

So when is the probability $\geq 1/2$ that 2 have the same birthday?

\[ 1 - s_{23} > 1 - 0.4973 = 0.5027 > 1/2 \]
Data mining

Data mining — techniques for finding patterns in collections of data.

Examples?
Data mining — techniques for finding patterns in collections of data.

Examples?

- marketing
- investment analysis
- quality control
- loan risk management
- fraud detection
- identifying functions of particular genes (from DNA)
Data mining

Done on static data collections — data warehouses.

Use a snapshot of the database.
Data mining

Common forms:

- **class description** — identifying properties that characterize a given group of data items (who buys small cars)

- **class discrimination** — identifying techniques that could be used to distinguish between groups (tell if current customer would buy a large car or a small)

- **cluster analysis** — finding groupings (people who see children's films have ages 4–10 and 25–40?)

- **association analysis** — finding links between data groups (people who buy pasta sauce also buy pasta)

- **outlier analysis** — finding data points which look wrong (credit card fraud)

- **sequential pattern analysis** — identifying patterns over time (climate patterns)
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Techniques:

- statistics
- database technology — giving data warehouses capability of presenting data as data cubes
  (data viewed from multiple perspectives — dimensions)
Data mining

Ethical and societal questions:

Is it OK that a store finds out that people who buy candy also buy chips and put them far apart?

Is it OK to find out and make public characteristics of people who commit crimes?