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4. Look in your textbook or my slides (or notes on the course’s homepage) for how to do things (page 239 for repeat loop, for example).
5. Don’t write more than necessary, but always explain, so it’s easy to see that your answer is correct and you understand it.
6. If given a choice, don’t choose to do the hard problem wrong.
7. Read all comments, even if it was approved. Correct all errors if it was not.
8. Submit redo via Blackboard. Give your TA your correct original.
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How do you sort? Think about cards.
Procedure Sort(List):
{ Input: List is a list }
{ Output: List, with same entries, but in nondecreasing order }

\[ N := 2 \]
while \( (N \leq \text{length}(\text{List})) \)
begin
\begin{align*}
\text{Pivot} & := \text{Nth entry} \\
\text{j} & := N - 1 \\
\text{while (j > 0 and jth entry > Pivot)} \\
\text{begin} \\
& \hspace{1em} \text{move jth entry to loc. j + 1} \\
& \hspace{2em} \text{j} := j - 1 \\
\text{end} \\
\text{place Pivot in j + 1st loc.} \\
\text{N} & := N + 1 \\
\end{align*}
end
Insertion Sort

What happens if List has 0 or 1 entry?

A. Sort crashes  
B. Sort returns the input list unchanged  
C. Sort returns something wrong

Vote at m.socrative.com. Room number 415439.
**Insertion Sort**

<table>
<thead>
<tr>
<th></th>
<th>17</th>
<th>8</th>
<th>15</th>
<th>53</th>
<th>18</th>
<th>12</th>
<th>2</th>
<th>75</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

$N$: 2

Pivot: 8

$j$: 1

$j$th entry: 17
## Insertion Sort

<table>
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<td>8</td>
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</table>

$N$: 2  

Pivot: 8  

$j$: 0  

$j$th entry: none
Insertion Sort

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<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

$N$: 3  
Pivot: 15  
$j$: 2  
$j$th entry: 17
Insertion Sort

\[
\begin{array}{cccccccc}
8 & 17 & 15 & 53 & 3 & 12 & 2 & 75 \\
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
\end{array}
\]

\(N\): 3  \\
Pivot: 15  \\
\(j\): 2  \\
\(j\)th entry: 17  \\
Continue on board.
Insertion Sort — correctness

procedure Sort(List):
  { Input: List is a list }
  { Output: List, with same entries, but in nondecreasing order }
  $N := 2$
  while ($N \leq \text{length}(\text{List})$)
  begin
    { loop invariants: 
      1. entries 1 thru $N - 1$ in List are in sorted order
      2. the same items are in List as originally }
    Pivot := $N$th entry
    $j := N - 1$
    while ($j > 0$ and $j$th entry $> $Pivot) begin
      move $j$th entry to loc. $j + 1$
      $j := j - 1$ end
    place Pivot in $j + 1$st loc.
    $N := N + 1$
  end
Insertion Sort — correctness

procedure Sort(List):
{ Input: List is a list }
{ Output: List, with same entries, but in nondecreasing order }

N := 2
while (N ≤ length(List)) begin
   Pivot := Nth entry
   j := N − 1
   while (j > 0 and jth entry > Pivot) begin
   { loop invariants: 1. no item in loc. j + 1
      2. entries in locs. j + 2 to N are larger than Pivot
      3. entries in locs. 1 to N − 1 stay in same relative order
      4. no entries in locs. N + 1 to length(List) are changed } 
      move jth entry to loc. j + 1
      j := j − 1  end
   place Pivot in j + 1st loc.
   N := N + 1
end
Insertion Sort — analysis

Suppose list has $n$ entries.
How many comparisons occur in the best case?

A. 1  
B. 2  
C. $n-1$  
D. $n$  
E. $n+1$

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(What is the best case?)
Insertion Sort — analysis

Worst case number of comparisons:

Outer loop from 2 to $n$. 

Insertion Sort

procedure Sort(List):
{ Input: List is a list }
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Pivot := \( N \)th entry
\[ j := N - 1 \]
while \( j > 0 \) and \( j \)th entry > Pivot) begin
\[ j := j - 1 \]
end
place Pivot in \( j + 1 \)st loc.
\[ N := N + 1 \]
end
Insertion Sort — analysis

Worst case number of comparisons:

Outer loop from 2 to \( n \).
Inner loop from \( N - 1 \) to 1.

Total: \( \sum_{N=2}^{n} (N - 1) = \sum_{i=1}^{n-1} i = \frac{n(n-1)}{2} \in \Theta(n^2) \).
Insertion Sort — analysis

Worst case number of comparisons:

Outer loop from 2 to $n$.

Inner loop from $N - 1$ to 1.

Total: $\sum_{N=2}^{n} (N - 1) = \sum_{i=1}^{n-1} i = \frac{n(n-1)}{2} \in \Theta(n^2)$.

Can it take this many comparisons?
Worst case number of comparisons:

Outer loop from 2 to $n$.
Inner loop from $N - 1$ to 1.

Total: $\sum_{N=2}^{n} (N - 1) = \sum_{i=1}^{n-1} i = \frac{n(n-1)}{2} \in \Theta(n^2)$.

Can it take this many comparisons?
Yes.
What list gives this worst case?

A. an ordered list
B. a list in reverse order
C. a random list
D. none of the above
E. all of the above

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Worst case number of comparisons:

Outer loop from 2 to $n$.

Inner loop from $N - 1$ to 1.

Total: $\sum_{N=2}^{n} (N - 1) = \sum_{i=1}^{n-1} i = \frac{n(n-1)}{2} \in \Theta(n^2)$.

Average case number of comparisons:

On average place next Pivot half way down the list.

$\frac{n(n-1)}{4} \in \Theta(n^2)$. 
Insertion Sort — analysis

Worst case number of comparisons:

Outer loop from 2 to \( n \).
Inner loop from \( N - 1 \) to 1.
Total: \( \sum_{N=2}^{n} (N - 1) = \sum_{i=1}^{n-1} i = \frac{n(n-1)}{2} \in \Theta(n^2) \).

Average case number of comparisons:

On average place next Pivot half way down the list.
\( \frac{n(n-1)}{4} \in \Theta(n^2) \).

There exist algorithms which do \( \Theta(n \log n) \) comparisons.
Classical bin packing

Use as few bins as possible:
Item sizes: \( n \times [1/2, \epsilon] \)
Bin size: 1

Result by First-Fit algorithm:
Classical bin packing

Use as few bins as possible:
Item sizes: $n \times \left[1/2, \varepsilon \right]$
Bin size: 1

Result by Worst-Fit algorithm:
Dual bin packing

Given a fixed number of bins, pack as many items as possible.

Bin size: 1
Number of bins: 4
Item sizes:

- $\frac{1}{4}, \frac{1}{4}, \frac{1}{4}$
- $\frac{5}{12}, \frac{1}{3}$
- $\frac{5}{12}, \frac{1}{3}$
- $\frac{5}{12}, \frac{1}{3}$
- $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$

Can they all be there?
Dual bin packing

Item sizes:

- $\frac{1}{4}, \frac{1}{4}, \frac{1}{4}$
- $\frac{5}{12}, \frac{1}{3}$
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Can they all be there?

First check:

$3 \frac{3}{12} + 3 \frac{9}{12} + 3 \frac{4}{12} = 4$
Dual bin packing

Item sizes:

- $\frac{1}{4}, \frac{1}{4}, \frac{1}{4}$
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Can they all be there?

What about First-Fit?

An optimal algorithm?
Bin packing

First-Fit is an on-line algorithm:
It handles requests without looking at future requests.
Some problems are on-line in nature. Examples?
Solving bin packing optimally is NP-hard.
Brute force takes a long time.
First-Fit is an **on-line** algorithm:
It handles requests without looking at future requests.
Some problems are on-line in nature. Examples?

Solving bin packing optimally is **NP-hard**.
Brute force takes a long time.
Approximation algorithms: First-Fit-Decreasing, even better...

Special case: all sizes multiples of $\frac{1}{12}$.
Fill one bin completely if possible.
First-Fit for dual bin packing

procedure First-Fit-Dual(List):
{ Input: List is a list of items with sizes \( \leq 1 \) }
{ Output: Number of rejected items }

    \( k := \) number of bins { all empty }
    Count := 0 { number rejected }
    get next item \( x \) and remove from list
    \( i := 1 \)
    while (\( i \leq k \) and \( x \) does not fit in bin \( i \))
        \( i := i + 1 \)
    if (\( i \leq k \))
        then put \( x \) in bin \( i \)
    else Count := Count + 1

return(Count)
First-Fit for dual bin packing (correct)

procedure First-Fit-Dual(List):
{ Input: List is a list of items with sizes $\leq 1$ }
{ Output: Number of rejected items }

$k := \text{number of bins} \begin{array}{l}
\text{\{ all empty \}}
\end{array}$

Count := 0 \begin{array}{l}
\text{\{ number rejected \}}
\end{array}

while there are still items in the list
begin
get next item $x$ and remove from list

$i := 1$

while ($i \leq k$ and $x$ does not fit in bin $i$)

$i := i + 1$

if ($i \leq k$)
then put $x$ in bin $i$
else Count := Count + 1

end

return(Count)