Institut for Matematik og Datalogi
Syddansk Universitet

February 4, 2004
JFB

On-Line Algorithms – F04 – Lecture 1

Textbook

Allan Borodin and Ran El-Yaniv, *Online Computation and Competitive Analysis*, Cambridge University Press, 1998. There will also be supplementary articles.

Format

The course will be taught by Joan Boyar. The lectures will be in English. Discussion sections will usually be on Mondays. The Wednesday class scheduled for the first week is cancelled. There will be an oral exam in June.

The weekly notes and other information about the course is available through the WorldWideWeb. Use the URL:

http://www.imada.sdu.dk/~joan/online/index.html.

Please do not hesitate to contact me if you have questions concerning the course. I have office hours on Mondays 13:00–13:45 and Thursdays 14:00–14:45.

Lecture, February 2

We begin with an introduction to the course. Then, we will cover chapter 1 in the textbook, probably covering up through section 1.3.

Lecture, February 11

We will continue with chapter 1 in the textbook.

Problems for Monday, February 9

1. Do Exercise 1.2 in the textbook.
2. For the proof of theorem 1.1, work out the details for insertion of an item.
3. This is a modification of Exercise 1.4. The algorithm *fractional MTF* uses a parameter $d \geq 1$. Let MTF_d be the variant of MTF that moves an accessed or inserted item at position i at least $\frac{i}{d} - 1$ positions closer to the front of the list. Show that $\mathrm{MTF}_d(\sigma) \leq d(2 \cdot \mathrm{OPT}_C + \mathrm{OPT}_A - n)$, where OPT_A is the total number of transpositions performed by OPT.

   Hint: Use d times the number of inversions for your potential function.
4. For the on-line Dual Bin Packing Problem, there are n bins of unit size. The request sequence is contains items of size $0 \leq s \leq 1$ which are to be placed in the bins. The goal is to accept and pack as many items in the request sequence as possible subject to the constraint that no bin should contain items whose sizes add up to more than 1. This is a maximization problem.

   - Give a definition of the comptetitive ratio for this problem which gives ratios of at most 1.
   - Consider the algorithm First-Fit, which places an item in the first bin in which it fits. First-Fit is a *fair* algorithm, an algorithm which never rejects an item if it fits in one of the bins when it is rejected. Define two more fair algorithms and one which is not fair.
   - Assume that $\frac{1}{k}$ is the size of the smallest item in any sequence. Prove an upper bound of $\frac{2-\frac{1}{k}}{k}$ on First-Fit's competitive ratio on sequence of this type (assume that OPT must also be fair). Is First-Fit competitive? Prove a lower bound of $\frac{1}{k}$ on First-Fit's competitive ratio on sequences of this type.