Institut for Matematik og Datalogi Syddansk Universitet March 12, 2004 JFB

On-Line Algorithms – F04 – Lecture 7

Lecture, March 10

We finished chapter 6 and began motivating the relative worst order ratio.

Lecture, March 17

We will begin looking at the article "The relative worst order ratio applied to paging", at http://www.imada.sdu.dk/~/online/paging.pdf. In section 2, we will initially only consider definitions 1 and 2 and skip the others. Next we will cover Lemmas 6 and 7 and Theorem 5, followed by Lemmas 3, 4, and 5 and Theorem 4. Then we will cover section 5.

Lecture, March 17

We will cover sections 3, 6 and 7. This will mean that we need to look at the other definitions in section 2, also.

Problems for Monday, March 15

- 1. Show that with the relative worst order ratio, for a given problem, the ordering as to which algorithms are better than which is transitive. Show that if $WR_{\mathbb{A},\mathbb{B}} \geq 1$ and $WR_{\mathbb{B},\mathbb{C}} \geq 1$, then $WR_{\mathbb{A},\mathbb{C}} \geq WR_{\mathbb{B},\mathbb{C}}$. Furthermore, show that if $WR_{\mathbb{A},\mathbb{B}}$ is bounded above by some constant, then $WR_{\mathbb{A},\mathbb{C}} \geq WR_{\mathbb{A},\mathbb{B}}$.
- 2. Lemma 4 in the article "The relative worst order ratio applied to paging" does not hold if the conservative algorithm is allowed look-ahead. How do you know this? Where does the proof fail?

- 3. Find another sequence which would separate LRU's and FWF's behavior under the relative worst order ratio. (It's not necessary to get as large a ratio as the one in the article. Try for $\frac{3}{2}$.)
- 4. Try defining an algorithm which is based on FIFO and uses look-ahead. What is its relative worst order ratio compared to FIFO? To LRU?
- 5. The proof of Lemma 1 only holds for $l \leq k$. Why? What if $l \geq k$?