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JFB

On-Line Algorithms – F05 – Lecture 6

Lecture, March 8

Kim Skak Larsen finished chapter 4 in the textbook.

Lecture, March 15

We will finish chapter 6 and may begin looking at the article "The relative worst order ratio applied to paging", at http://www.imada.sdu.dk/~joan/online/paging.pdf. In section 2, we will initially only consider definitions 1 and 2 and skip the others. Next we will cover Lemmas 6 and 7 and Theorem 5, followed by Lemmas 3, 4, and 5 and Theorem 4. Then we will cover section 5.

Lecture, March 29

We will continue looking at "The relative worst order ratio applied to paging".

Problems for March 21

1. Do Exercise 6.1.
2. (Part of Exercise 6.4.) Show that the algorithm PERM_π is neither a marking algorithm nor a conservative algorithm. Try using $N = k + 2$.
3. In the absent minded driver problem, is $\frac{1}{2}$ the optimal value for the behavioral strategy?
4. Do Exercise 6.5.
5. Do Exercise 6.6.

6. Consider the Dual Bin Packing Problem from the first weekly note, and assume we are only considering fair algorithms. Consider the following adversary against a deterministic algorithm $\mathbb{A}$: Give $\mathbb{A}$ the following request sequence, divided into three phases. Phase 1 consists of n small items of size $\frac{1}{n}$. Phase 2 consists of items, one for each bin which $\mathbb{A}$ did not fill completely with size equal to the empty space in that bin, sorted in decreasing order. After these are given, $\mathbb{A}$ has filled all bins completely and so must reject the items in Phase 3, which consists of $\frac{n^2}{4}$ items of size $\frac{1}{n}$.

   To analyze this, let q denote the number of bins in $\mathbb{A}$'s configuration which have at least 2 items after the first phase.

   In the case where $q < \frac{n}{4}$, we know that $\mathbb{A}$ has at least $n - q \geq \frac{3n}{4}$ bins with at most one item after Phase 1. OPT can arrange the items from Phase 1 such that half of the bins contain two items and half contain no items.

   In the case where $q \geq \frac{n}{4}$, we know that $\mathbb{A}$ has at least $\frac{n}{4}$ empty bins after Phase 1. OPT places each of the items from Phase 1 in a different bin.

   a. Use the adversary above to show that no fair algorithm for the dual bin packing problem is more than $\frac{8}{6+n}$-competitive (strict competitive ratio).

   b. Try changing the above adversary to an adaptive on-line adversary. What result can you get?

   c. Define an algorithm which uses a mixed strategy to solve the dual bin packing problem.

   d. Define an algorithm which uses a behavioral strategy to solve the dual bin packing problem.