Institut for Matematik og Datalogi
Syddansk Universitet

March 8, 2006
JFB

On-Line Algorithms – F06 – Lecture 7

Announcement

On the remaining Tuesdays, we will meet in U89 at 12:15.

Lecture, March 8

Kim Skak Larsen finished chapter 4.

Lecture, March 14

We will cover chapter 6 in the textbook.

Lecture, March 21

We will begin looking at the article "The relative worst order ratio applied to paging", at http://www.imada.sdu.dk/~joan/online/paging2.pdf. (See the course's homepage.) In section 2, we will initially only consider definitions 1 and 2 and skip the others. Next we will cover up through Theorem 5 of section 4, and then section 6.

Problems for March 14

1. Do Exercise 4.2 in the textbook.
2. Do Exercise 4.3 in the textbook (for $h = k$).
3. Do Exercise 4.5 in the textbook.
4. Do Exercise 4.6 in the textbook.

5. Consider an optimal offline paging algorithm. Find arbitrarily long request sequences with more than k pages for which it does not help OPT if it had more than k pages in its fast memory (i.e. OPT should have the same number of page faults with k pages as it would have with more pages).

6. Consider an algorithm with look-ahead s, meaning that when deciding what to do about the current page request, the algorithm can see the next s requests before deciding what to do.

   - Prove that any such deterministic algorithm has competitive ratio at least k.
   - Consider LRU(s), the algorithm which uses the LRU rule, ignoring (and never evicting) any page in the next s requests. Show that it does at least as well as LRU on any request sequence (assuming they start with the same pages in fast memory).

Problems for March 21

1. Do Exercise 6.1.

2. (Part of Exercise 6.4.) Show that the algorithm PERM_π is neither a marking algorithm nor a conservative algorithm. Try using $N = k + 2$.

3. In the absent minded driver problem, is $\frac{1}{2}$ the optimal value for the behavioral strategy?

4. Do Exercise 6.5.

5. Do Exercise 6.6.

6. Consider the Dual Bin Packing Problem from the first weekly note, and assume we are only considering fair algorithms. Consider the following adversary against a deterministic algorithm $\mathbb{A}$: Give $\mathbb{A}$ the following request sequence, divided into three phases. Phase 1 consists of n small items of size $\frac{1}{n}$. Phase 2 consists of items, one for each bin which $\mathbb{A}$ did not fill completely with size equal to the empty space in that bin, sorted in decreasing order. After these are given, $\mathbb{A}$ has filled all bins completely and so must reject the items in Phase 3, which consists of $\frac{n^2}{4}$ items of size $\frac{1}{n}$.

To analyze this, let q denote the number of bins in $\mathbb{A}$'s configuration which have at least 2 items after the first phase.

In the case where $q < \frac{n}{4}$, we know that $\mathbb{A}$ has at least $n - q \geq \frac{3n}{4}$ bins with at most one item after Phase 1. OPT can arrange the items from Phase 1 such that half of the bins contain two items and half contain no items.

In the case where $q \geq \frac{n}{4}$, we know that $\mathbb{A}$ has at least $\frac{n}{4}$ empty bins after Phase 1. OPT places each of the items from Phase 1 in a different bin.

a. Use the adversary above to show that no fair algorithm for the dual bin packing problem is more than $\frac{8}{6+n}$-competitive (strict competitive ratio).

b. Try changing the above adversary to an adaptive on-line adversary. What result can you get?

c. Define an algorithm which uses a mixed strategy to solve the dual bin packing problem.

d. Define an algorithm which uses a behavioral strategy to solve the dual bin packing problem.