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JFB

On-Line Algorithms – F06 – Lecture 8

Lecture, March 15

We covered chapter 6 in the textbook.

Lecture, March 22

We will begin looking at the article "The relative worst order ratio applied to paging", at http://www.imada.sdu.dk/~joan/online/paging2.pdf. (See the course's homepage.) In section 2, we will initially only consider definitions 1 and 2 and skip the others. Next we will cover up through Theorem 5 of section 4, and then section 6.

Lecture, March 29

We will cover sections 3 and 5, plus the definitions for relatedness and weakly comparable in section 2 of "The relative worst order ratio applied to paging".

Problems for March 21

1. Do Exercise 6.1.
2. (Part of Exercise 6.4.) Show that the algorithm PERM_π is neither a marking algorithm nor a conservative algorithm. Try using $N = k + 2$.
3. In the absent minded driver problem, is $\frac{1}{2}$ the optimal value for the behavioral strategy?
4. Do Exercise 6.5.
5. Do Exercise 6.6.

6. Consider the Dual Bin Packing Problem from the first weekly note, and assume we are only considering fair algorithms. Consider the following adversary against a deterministic algorithm $\mathbb{A}$: Give $\mathbb{A}$ the following request sequence, divided into three phases. Phase 1 consists of n small items of size $\frac{1}{n}$. Phase 2 consists of items, one for each bin which $\mathbb{A}$ did not fill completely with size equal to the empty space in that bin, sorted in decreasing order. After these are given, $\mathbb{A}$ has filled all bins completely and so must reject the items in Phase 3, which consists of $\frac{n^2}{4}$ items of size $\frac{1}{n}$.

   To analyze this, let q denote the number of bins in $\mathbb{A}$'s configuration which have at least 2 items after the first phase.

   In the case where $q < \frac{n}{4}$, we know that $\mathbb{A}$ has at least $n - q \geq \frac{3n}{4}$ bins with at most one item after Phase 1. OPT can arrange the items from Phase 1 such that half of the bins contain two items and half contain no items.

   In the case where $q \geq \frac{n}{4}$, we know that $\mathbb{A}$ has at least $\frac{n}{4}$ empty bins after Phase 1. OPT places each of the items from Phase 1 in a different bin.

   a. Use the adversary above to show that no fair algorithm for the dual bin packing problem is more than $\frac{8}{6+n}$-competitive (strict competitive ratio).

   b. Try changing the above adversary to an adaptive on-line adversary. What result can you get?

   c. Define an algorithm which uses a mixed strategy to solve the dual bin packing problem.

   d. Define an algorithm which uses a behavioral strategy to solve the dual bin packing problem.

Problems for March 28

1. Show that with the relative worst order ratio, for a given problem, the ordering as to which algorithms are better than which is transitive. Show that if $\mathrm{WR}_{\mathbb{A},\mathbb{B}} \geq 1$ and $\mathrm{WR}_{\mathbb{B},\mathbb{C}} \geq 1$, then $\mathrm{WR}_{\mathbb{A},\mathbb{C}} \geq \mathrm{WR}_{\mathbb{B},\mathbb{C}}$. Furthermore, show that if $\mathrm{WR}_{\mathbb{A},\mathbb{B}}$ is bounded above by some constant, then $\mathrm{WR}_{\mathbb{A},\mathbb{C}} \geq \mathrm{WR}_{\mathbb{A},\mathbb{B}}$.

2. Lemma 4 in the article "The relative worst order ratio applied to paging" does not hold if the conservative algorithm is allowed look-ahead. How do you know this? Where does the proof fail?

3. Find another sequence which would separate LRU's and FWF's behavior under the relative worst order ratio. (It's not necessary to get as large a ratio as the one in the article. Try for $\frac{3}{2}$.)

4. Try defining an algorithm which is based on FIFO and uses look-ahead. What is its relative worst order ratio compared to FIFO? To LRU?

5. Consider the algorithm for dual bin packing (fixed number of bins, maximizing the number of accepted items) behaves exactly as First-Fit would unless the item x is larger than $\frac{1}{2}$ and would be placed in the l ast bin, bin n. The algorithm FF_n rejects such an item and is thus not fair.

   Show that FF_n is better than FF, according to the relative worst order ratio.