Institut for Matematik og Datalogi
Syddansk Universitet

May 12, 2009
JFB

# On-Line Algorithms – F09 – Lecture 6

## Lecture, May 6

Kim Skak Larsen finished chapter 4 and I finished through Example 6.6 in chapter 6 of the textbook.

## Lecture, May 11

After finishing the exercises, I finished up through Lemma 6.2 of chapter 6.

## Lecture, May 13

We will finish chapter 6 and then begin looking at the paper, "The relative worst order ratio applied to paging", by J. Boyar, L.M. Favrholdt, and K.S. Larsen, in *Journal of Computer and System Sciences*, volume 73, pages 817–843, 2007. You get this through the electronic journals SDU's library has. In section 2, we will initially only consider definitions 1 and 2 and skip the others. Next we will cover up through Corollary 3 of section 4, and then section 6.

## Lecture, May 18

We will cover sections 3, the definitions for relatedness and weakly comparable in section 2 of "The relative worst order ratio applied to paging", and Theorem 7 of section 5.

## Problems for May 20

1. Do Exercise 6.1.

2. (Part of Exercise 6.4.) Show that the algorithm $\mathrm{PERM}_\pi$ is neither a marking algorithm nor a conservative algorithm. Try using $N = k + 2$.

3. In the absent minded driver problem, is $\frac{1}{2}$ the optimal value for the behavioral strategy?

4. Do Exercise 6.5.

5. Do Exercise 6.6.

6. Consider the Dual Bin Packing Problem from the first weekly note, and assume we are only considering fair algorithms. Consider the following adversary against a deterministic algorithm $\mathbb{A}$: Give $\mathbb{A}$ the following request sequence, divided into three phases. Phase 1 consists of $n$ small items of size $\frac{1}{n}$. Phase 2 consists of items, one for each bin which $\mathbb{A}$ did not fill completely with size equal to the empty space in that bin, sorted in decreasing order. After these are given, $\mathbb{A}$ has filled all bins completely and so must reject the items in Phase 3, which consists of $\frac{n^2}{4}$ items of size $\frac{1}{n}$.

   To analyze this, let $q$ denote the number of bins in $\mathbb{A}$'s configuration which have at least 2 items after the first phase.

   In the case where $q < \frac{n}{4}$, we know that $\mathbb{A}$ has at least $n - q \geq \frac{3n}{4}$ bins with at most one item after Phase 1. OPT can arrange the items from Phase 1 such that half of the bins contain two items and half contain no items.

   In the case where $q \geq \frac{n}{4}$, we know that $\mathbb{A}$ has at least $\frac{n}{4}$ empty bins after Phase 1. OPT places each of the items from Phase 1 in a different bin.

   a. Use the adversary above to show that no fair algorithm for the dual bin packing problem is more than $\frac{8}{6+n}$-competitive (strict competitive ratio).
   b. Try changing the above adversary to an adaptive on-line adversary. What result can you get?
   c. Define an algorithm which uses a mixed strategy to solve the dual bin packing problem.
   d. Define an algorithm which uses a behavioral strategy to solve the dual bin packing problem.