Institut for Matematik og Datalogi
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JFB

On-Line Algorithms – F09 – Lecture 8

Lecture, May 18

We covered sections 3 and 6, the definitions for relatedness and weakly comparable in section 2 of "The relative worst order ratio applied to paging", and Theorem 7 of section 5.

Lecture, May 22

We will cover Theorem 8 in section 5 of "The relative worst order ratio applied to paging", and chapters 7 and 8 in the textbook quickly.

Lecture, May 27

We will cover sections 10.1 through 10.4 and 10.6 of chapter 10 in the textbook.

Problems for May 29

1. Work out an example showing how to change a worst case ordering for LRU to a worst case ordering for PERM_π.

2. Compare First-Fit and Worst-Fit for the classical bin packing problem (trying to minimize the number of bins used). Worst-Fit is the algorithm which places an item in the most empty open (already used) bin, if it fits in any open bin. Otherwise it opens a new bin.

3. If we get this far in lecture, do exercise 7.3 in the textbook. See page 122 for the coupon collector's problem. Assume that there are $k + 1$ pages in all, and obtain kH_k as a result.

4. Work out Example 8.5, and apply Yao's principle correctly to Example 8.4 in the textbook, using the distribution given there. You will not get as good a result for Example 8.4 as for Example 8.5.

5. Consider the following on-line problem: We have one processor. Jobs arrive over time; job J_j with processing time p_j arrives at time r_j. A job can be assigned to run on the processor when it arrives or any time after that. It can also be started on the processor, stopped at some point, and restarted at some later point. No two jobs may be running at the same time. The goal is to minimize total completion time. Let C_j denote the completion time of job j. The total completion time is $\sum C_j$.

   Use Yao's principle to prove a lower bound on the competitive ratio of any randomized algorithm for this problem. Consider the following probability distribution on request sequences: At time 0, a job with processing time 1 arrives. At time $\frac{1}{3}$, two jobs with processing time 0 arrive. At time 1, all of the following jobs arrive with probability p (with probability $1 - p$ none of them arrive): 10 jobs with processing time 0 and four jobs with processing time 1.

6. Can Yao's principle be applied to the relative worst order ratio?