Institut for Matematik og Datalogi
Syddansk Universitet

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JFB

On-Line Algorithms – F14 – Lecture 1

Textbook

Allan Borodin and Ran El-Yaniv, *Online Computation and Competitive Analysis*, Cambridge University Press, 1998. There is a supplementary article; see the course homepage.

Format

The course will be taught by Joan Boyar. The lectures will be in English. There will be an oral exam on March 31.

The weekly notes and other information about the course is available through Blackboard or the URL:

http://www.imada.sdu.dk/~joan/online/

Please do not hesitate to contact me if you have questions concerning the course. I have office hours on Mondays from 13:00 to 13:45 and on Fridays from 8:30 to 9:15.

Lecture, February 3

We will begin with an introduction to the course. Then, we will begin on chapter 1 in the textbook, probably covering up through section 1.4.

Lecture, February 5

We will continue with chapter 1 in the textbook.

Problems for February 6

Note that we may not finish all of these problems. If not, we will continue on February 12.

1. Do Exercise 1.2 in the textbook (assume the static problem).

2. For the proof of Theorem 1.1, work out the details for insertion of an item.

3. This is a modification of Exercise 1.4. The algorithm *fractional MTF* uses a parameter $d \geq 1$. Let MTF_d be the variant of MTF that behaves as follows. When an item is accessed or inserted at position i, it moves it $\frac{i}{d}-1$ positions closer to the front of the list. Show that $\mathrm{MTF}_d(\sigma) \leq d(2 \cdot \mathrm{OPT}_C + \mathrm{OPT}_A - n)$, where OPT_A is the total number of transpositions performed by OPT.

   Hint: Use d times the number of inversions for your potential function.

4. Do Exercise 1.6 in the textbook.

5. Consider the algorithm MTF2 for list accessing, which moves an item to the front of the list every second time it accesses it. Let the list consist of the items $\{a_1, a_2, ..., a_n, b_1, b_2, ..., b_n\}$. Show that on the following sequence (repeated s times), the ratio of MTF2's performance to OPT's is at least $\frac{7}{3}$.

$$\begin{array}{ll} \langle a_1, a_2, ..., a_n, & b_n, b_n, b_{n-1}, b_{n-1}, ..., b_1, b_1, \\ a_n, a_{n-1}, ..., a_1, & b_n, b_n, b_{n-1}, b_{n-1}, ..., b_1, b_1, \\ b_1, b_2, ..., b_n, & a_n, a_n, a_{n-1}, a_{n-1}, ..., a_1, a_1, \\ b_n, b_{n-1}, ..., b_1, & a_n, a_n, a_{n-1}, a_{n-1}, ..., a_1, a_1\rangle \end{array}$$

   Hint: Assume that MTF2 starts with the list ordered as

$$\langle a_1, a_2, ..., a_n, b_1, b_2, ..., b_n\rangle$$

   while the algorithm it is competing against starts with

$$\langle a_1, a_2, ..., a_n, b_n, b_{n-1}, ..., b_1\rangle$$

   Note that this observation is due to Ansgar Grüne and his students and reveals an error in Exercise 1.5 in the textbook.

6. Show that MTF2 is 3-competitive using the potential function method. This can be done counting inversions, as for MTF, but giving the inversions weights based on how many requests to the item are necessary before MTF2 moves it to the front (the weights are 1 or 2). Then the two cases of whether or not MTF2 moves an item to the front are handled separately in the amortized analysis.

7. For the on-line Dual Bin Packing Problem, there are n bins of unit size. The request sequence contains items of sizes $0 \leq s \leq 1$ which are to be placed in the bins. The goal is to accept and pack as many items in the request sequence as possible subject to the constraint that no bin should contain items whose sizes add up to more than 1. This is a maximization problem.

   (a) Give a definition of the competitive ratio for this problem which gives ratios of at most 1.

   (b) Consider the algorithm First-Fit, which places an item in the first bin in which it fits. First-Fit is a *fair* algorithm, an algorithm which never rejects an item if it fits in one of the bins when it is rejected. Define two more fair algorithms and one which is not fair.

   (c) Assume that $\frac{1}{k}$ is the size of the smallest item in any sequence. Prove an upper bound of $\frac{2-\frac{1}{k}}{k}$ on First-Fit's competitive ratio on sequences of this type (assume that OPT must also be fair). Is First-Fit competitive? Prove a lower bound of $\frac{1}{k}$ on First-Fit's competitive ratio on sequences of this type.