

## On-Line Algorithms – F14 – Lecture 9

### **Lecture, March 12**

We covered section 3 (RLRU), the definitions for relatedness and weakly comparable in section 2 and Theorem 7 of section 5 of “The relative worst order ratio applied to paging”. (Note that the last problem for March 17 should be ignored.) We started on chapter 7 in the textbook.

### **Lecture, March 13**

We briefly covered chapters 7 and 8 in the textbook, with most emphasis being on the proof of the lower bound for randomized paging using Yao’s principle.

### **Lecture, March 20**

We will cover sections 10.1 and 10.4 of chapter 10 in the textbook (if we don’t do this on March 17 after discussing the problems). We will start on chapter 12.

### **Lecture, March 24**

We will cover sections 12.1 and 12.2 in the textbook.

### **Problems for March 26**

1. (Easy) Show that the makespan problem for identical machines is NP-hard.

2. Suppose that GREEDY is allowed  $n$  identical machines, while OPT is only allowed to use  $m < n$  machines. Give a sequence showing that the ratio of GREEDY's performance to OPT's can be at least  $1 + \frac{m-1}{n}$  for the makespan problem. Then show that GREEDY can always achieve this ratio against such a bounded OPT.
3. Consider remark 12.1 on page 208. What is meant here? Why is there no problem if the loads can be greater than 1? (Do not try to prove the desired result for loads of at most 1.)
4. Define POST-GREEDY with release dates as the algorithm which assigns a new job (given at its release date) to the first processor which becomes free. (Jobs have processing times which may be unknown, and only one job may be running on a processor at a time. There are  $m$  processors.) Show that POST-GREEDY is  $(2 - \frac{1}{m})$ -competitive.
5. Consider the algorithm for dual bin packing (fixed number of bins, maximizing the number of accepted items) behaves exactly as First-Fit would unless the item  $x$  is larger than  $\frac{1}{2}$  and would be placed in the last bin, bin  $n$ . The algorithm  $FF_n$  rejects such an item and is thus not fair.

Show that  $FF_n$  is better than FF, according to the relative worst order ratio.