Institut for Matematik og Datalogi
Syddansk Universitet

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JFB

Online Algorithms – F17 – Assignment 2

Assignment due Wednesday, April 19, 12:15

Note that this is part of your exam project, so it must be approved in order for you to take the exam in June, and you may not work with others not in your group. If it is late, it will not be accepted (though it could become an assignment you redo). You may work in groups of two (or three). Turn in the assignment through the SDU Assignment system in Blackboard, and remember to keep your receipt. Turn in one PDF file per group.

1. Consider an artificial game, called asymmetric string guessing. This vaguely resembles Example 6.1 in the textbook, but is quite different is some respects. There are two players, the presenter and the guesser. The presenter has a binary string of length n, $\langle x_1, x_2, ..., x_n\rangle$ which is unknown to the guesser. The presenter starts and presents n. The guesser then makes a guess $y_1 \in \{0, 1\}$ for x_1. Now it is the presenter's turn, and it presents the actual value for x_1. This continues for each bit, so in round i, the presenter reveals x_{i-1}, the guesser chooses $y_i \in \{0, 1\}$. The game ends with the presenter revealing x_n and the guesser doing nothing. The score is $+1$ for the guesser for each value i where $y_i = x_i$, 0 for each i where $y_i = 1$ and $x_i = 0$, and the score is $-\infty$ for each i where $y_i = 0$ and $x_i = 1$. Thus, if the string $\langle x_1, x_2, x_3, x_4, x_5\rangle = \langle 0, 1, 1, 0, 1\rangle$ and $\langle y_1, y_2, y_3, y_4, y_5\rangle = \langle 1, 1, 1, 0, 1\rangle$, then there were four correct guesses giving value 4 for the guesser (since the wrong guess was of the form $x_1 = 0$ and $y_1 = 1$, there is no penalty for that). However, if the guess was $\langle 0, 0, 1, 0, 1\rangle$, there are still four correct guesses, but the wrong guess is of the form $x_2 = 1$ and $y_2 = 0$, so the value is $-\infty$ for the guesser.

   (a) Show the game in extensive (or strategic) form for $n = 2$.

(b) What pure stragegies does the guesser have for $n = 2$? List them all, along with the value for the guesser for each possible string $\langle x_1, x_2 \rangle$.

(c) One can consider this game as an online problem with the online algorithm playing the part of the guesser. Assume that the input with $\langle x_1, x_2, ..., x_n \rangle = \langle 0, 0, ..., 0 \rangle$ is not allowed.

- What is the best deterministic online algorithm for this problem? What is its competitive ratio? Prove this.
- Use Yao's principle to prove that no randomized algorithm can do better than this.

2. Consider the paging algorithm 2ndLRU, from the first assignment: On a page fault, 2ndLRU evicts the page with the least recent second to last request. If there are pages in cache which have been requested fewer than two times, then the least recently used page among those only requested once is evicted. For example, with cache size 3 and request sequence $\langle p_1, p_2, p_3, p_3, p_1, p_2, p_4 \rangle$, on the last request, 2ndLRU evicts page p_1, whereas LRU would have evicted p_3.

   In comparing 2ndLRU to to LRU using relative worst order analysis, one could start by showing that LRU can be a factor $\frac{k+1}{2}$ worse than 2ndLRU by using a family of sequences similar to those used to separate RLRU from LRU. Prove this subresult for relative worst order analysis of 2ndLRU and LRU.

3. We know that RLRU is neither a conservative algorithm nor a marking algorithm, due to a specific sequence. Various results we have seen regarding relative worst order analysis also show this. Which ones and how?

4. Compare MTF and TRANS for the list access problem, using relative worst order analysis. You may assume that MTF is at least as good as TRANS and just prove a separation of $\frac{\ell}{2}$, where ℓ is the length of the list.